Lecture 2: **Intermediate macroeconomics, autumn 2012**

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Literature: ***Mankiw, Chapters 3, 7 and 8.***

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Topics

- **Production**
- **Labour productivity and economic growth**
- **The Solow Model**
- **Endogenous growth**

$$Y = F(K, L)$$

$$MPL = F(K, L + 1) - F(K, L)$$

$$MPL = \frac{dY}{dL} = \frac{dF(K, L)}{dL} = F_L$$

$$MPK = F(K + 1, L) - F(K, L)$$

$$MPK = \frac{dY}{dK} = \frac{dF(K, L)}{dK} = F_K$$

Figure 3-3: The production function

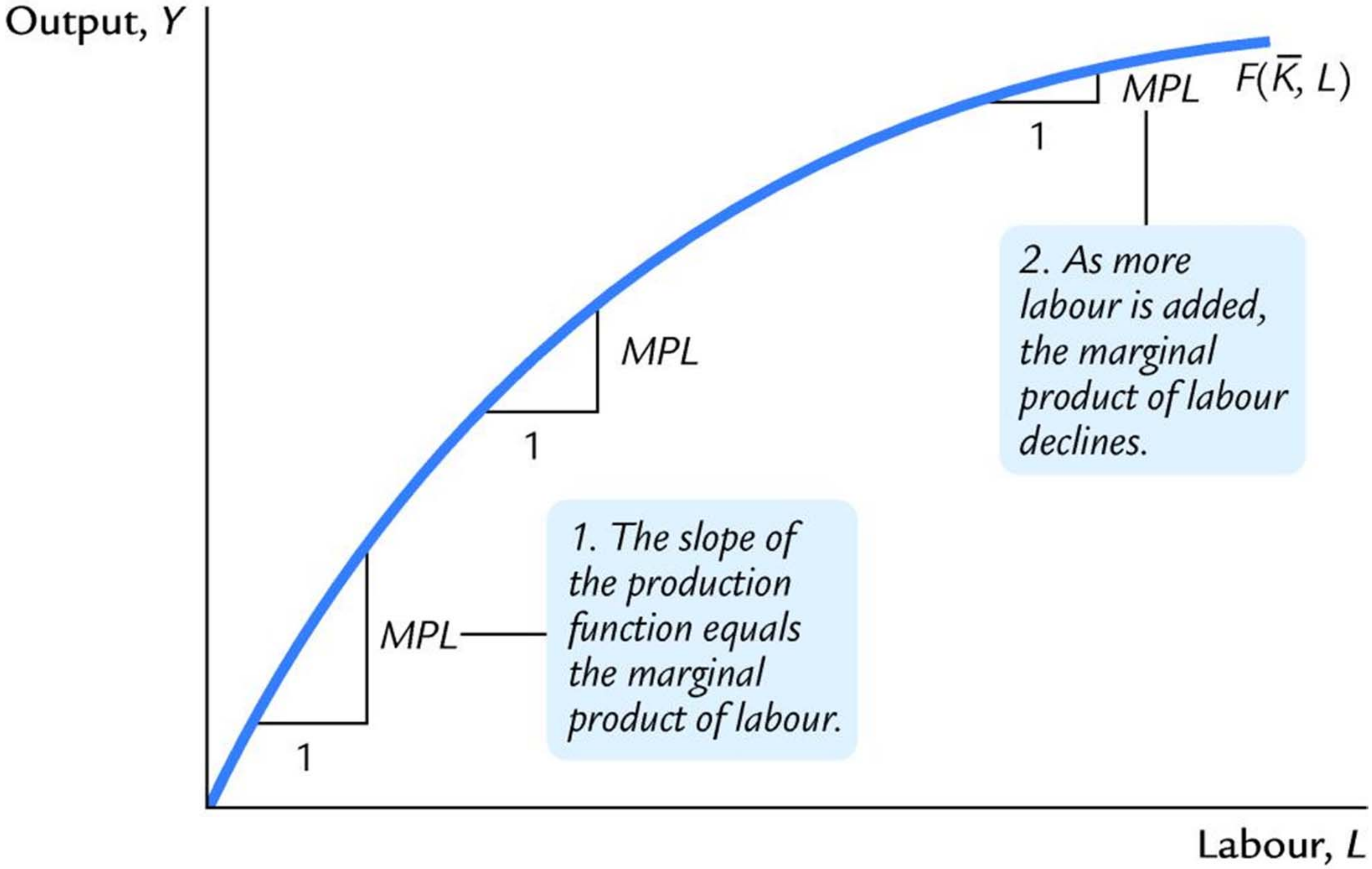
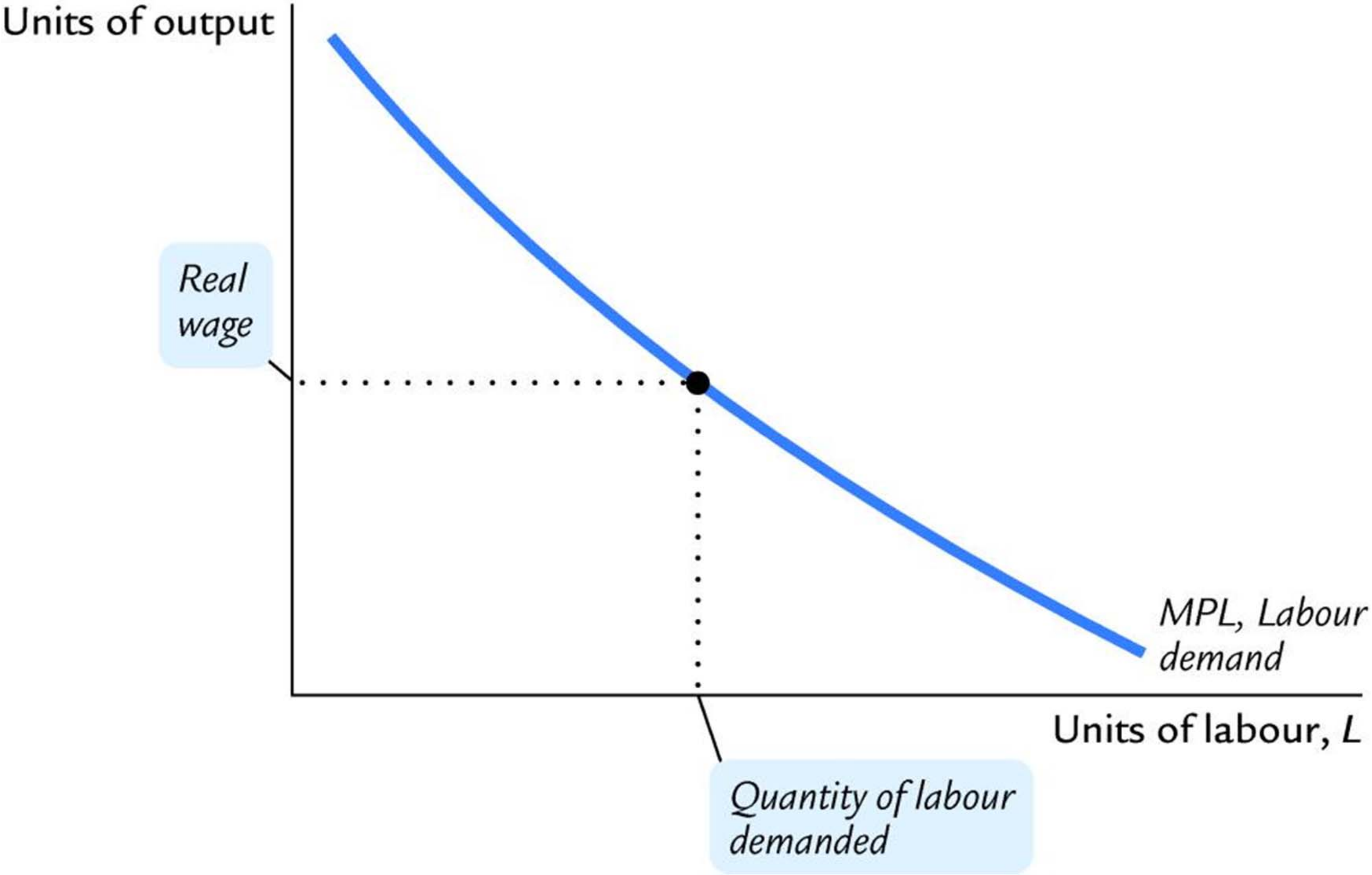


Figure 3-4: The marginal product of labour schedule



Profit maximisation

General: suppose *y* = *f* (*x*, *z*). The first-order conditions (FOCs) for maximum of *y* are:

$$\frac{dy}{dx} = f_x = 0$$

$$\frac{dy}{dx} = f_z = 0$$

Profit maximisation

$$\pi = PY - RK - WL = PF(K, L) - RK - WL$$

$$\frac{d\pi}{dL} = PF_L - W = 0 \Leftrightarrow F_L = \frac{W}{P}$$

$$MPL = \frac{W}{P}$$

$$\frac{d\pi}{dK} = PF_K - R = 0 \Leftrightarrow F_K = \frac{R}{P}$$

$$MPK = \frac{R}{P}$$

<u>Production function</u>

$Y = AF(K, L)$ $\qquad$ A = total factor productivity

It holds that:

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

α = capital income share
1-α = labour income share

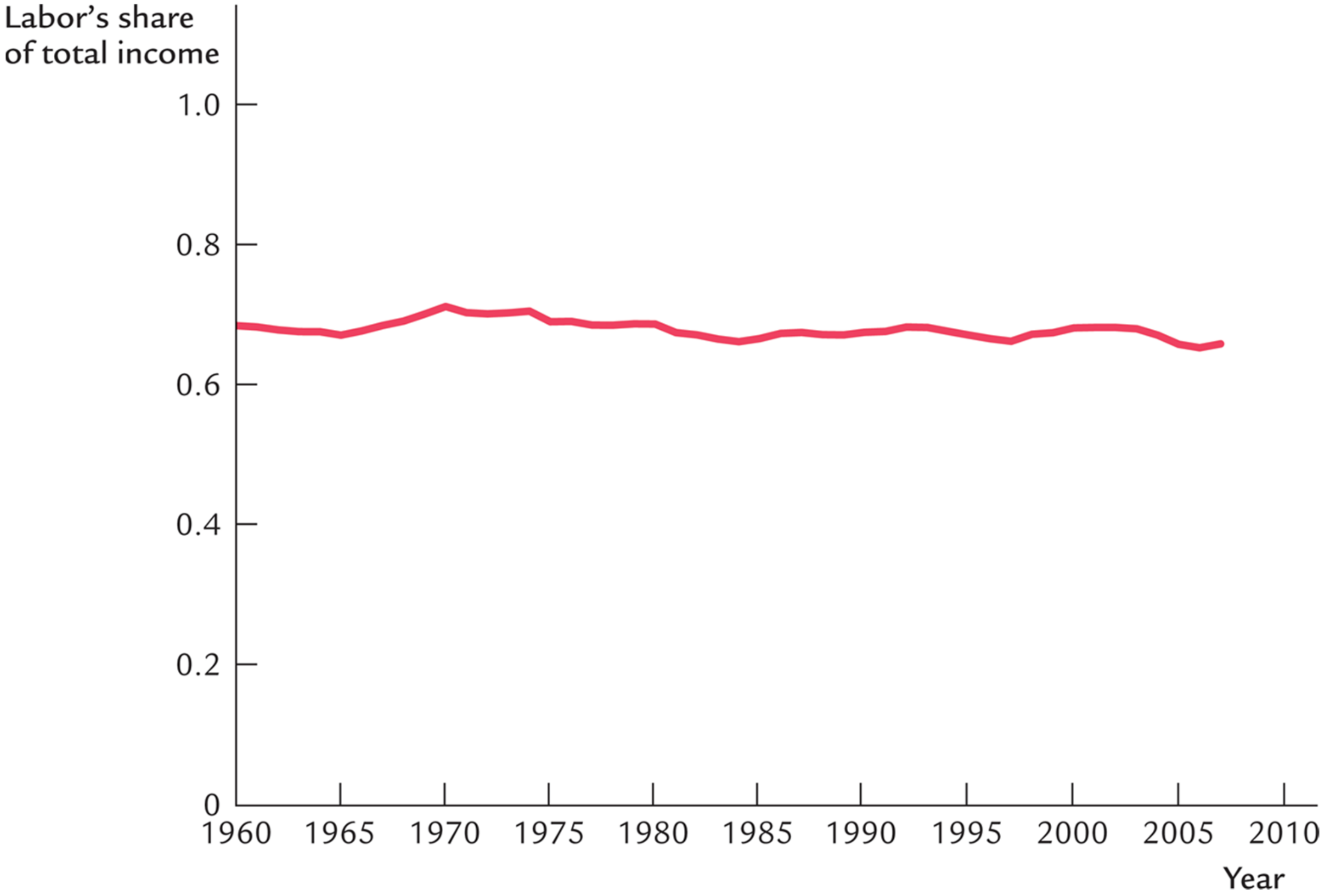
GDP growth = total factor productivity growth
+ contribution from growth of the capital stock
+ contribution from growth of the labour force

<u>Growth accounting</u>

The Solow-residual:

$$\frac{\Delta \mathrm{A}}{A} \approx \frac{\Delta Y}{Y} - \alpha\frac{\Delta K}{K} - (1-\alpha)\frac{\Delta L}{L}$$

<u>Figure 3-5: The ratio of labour income to total income</u>



Mathematical preliminaries: the natural logarithm

Recall that $\ell n\ x$ is the natural logarithm of x. By definition:

$$x = e^{a} \Leftrightarrow a = \ell n\ x$$

Properties:

$$\ell n\ (xy) = \ell n\ x + \ell n\ y$$
$$\ell n \left(\frac{x}{y}\right) = \ell n\ x - \ell n\ y$$
$$\ell n\ x^{\beta} = \beta\ \ell n\ x$$

Rules of differentiation

If $y = f(g)$ and $g = g(x)$ so that

$$y = f(g(x))$$

then

$$\frac{dy}{dx} = \frac{\partial f}{\partial g}\frac{dg}{dx} = f_g g_x \qquad (1)$$

Moreover, the derivative of the ℓn-function is given by:

$$\frac{d(\ell n\, x)}{dx} = \frac{1}{x} \qquad (2)$$

and for polynomials:

$$\frac{d(x^{\gamma})}{dx} = \gamma x^{\gamma-1}$$

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Cobb-Douglas production function

$Y = AF(K, L) = AK^{\alpha}L^{1-\alpha}$

K, L and A and thus also Y are functions of time (continuous-time formulation).

$\because \quad Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}$

Taking logarithms:

$$\ell n\, Y(t) = \ell n\, A(t) + \ell n\, K(t)^{\alpha} + \ell n\, L(t)^{1-\alpha}$$
$$\ell n\, Y(t) = \ell n\, A(t) + \alpha\, \ell n\, K(t) + (1-\alpha)\, \ell n\, L(t)$$

Differentiation w.r.t. time gives:

$$\frac{d\ell n\, Y(t)}{dt} = \frac{d\ell n\, A(t)}{dt} + \alpha \frac{d\ell n\, K(t)}{dt} + (1-\alpha)\frac{d\ell n\, L(t)}{dt}$$

$$\frac{dY}{dt} \bullet \frac{1}{Y} = \frac{dA}{dt} \bullet \frac{1}{A} + \alpha \frac{dK}{dt} \bullet \frac{1}{K} + (1-\alpha)\frac{dL}{dt} \bullet \frac{1}{L}$$

Call $\frac{dY}{dt} = \dot{Y}$, $\frac{dA}{dt} = \dot{A}$, $\frac{dK}{dt} = \dot{K}$ och $\frac{dL}{dt} = \dot{L}$

$$\because \quad \frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha \frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}$$

α = profit share
1-α = wage share

The discrete-time equivalent is:

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

$\Delta Y = Y_t - Y_{t-1}$ etc.

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<u>Profit maximisation with Cobb-Douglas production function</u>

$$\pi = PY - RK - WL = PAK^{\alpha}L^{1-\alpha} - RK - WL$$

$$\frac{d\pi}{dL} = (1-\alpha)PAK^{\alpha}L^{-\alpha} - W = 0$$

$$MPL = (1-\alpha)AK^{\alpha}L^{-\alpha} = \frac{W}{P}$$

$$MPL = \frac{(1-\alpha)AK^{\alpha}L^{-\alpha} \cdot L}{L} = \frac{W}{P}$$

$$MPL = \frac{(1-\alpha)AK^{\alpha}L^{1-\alpha}}{L} = \frac{W}{P}$$

$$MPL = (1-\alpha)\frac{Y}{L} = \frac{W}{P}$$

$$1-\alpha = \frac{WL}{PY} = \text{the labour share}$$

TABLE 3-1

Growth in Labor Productivity and Real Wages: The U.S. Experience

Time Period	Growth Rate of Labor Productivity	Growth Rate of Real Wages
1959–2007	2.1%	2.0%
1959–1973	2.8	2.8
1973–1995	1.4	1.2
1995–2007	2.5	2.4

Source: *Economic Report of the President 2008*, Table B-49, and updates from the U.S. Department of Commerce website. Growth in labor productivity is measured here as the annualized rate of change in output per hour in the nonfarm business sector. Growth in real wages is measured as the annualized change in compensation per hour in the nonfarm business sector divided by the implicit price deflator for that sector.

TABLE 7-1

International Differences in the Standard of Living

Country	Income per person (2007)	Country	Income per person (2007)
United States	$45,790	Indonesia	3,728
Japan	33,525	Philippines	3,410
Germany	33,154	India	2,753
Russia	14,743	Vietnam	2,600
Mexico	12,780	Pakistan	2,525
Brazil	9,570	Nigeria	1,977
China	5,345	Bangladesh	1,242

Source: The World Bank.

Constant returns to scale

Y = F(K, L)

zY = zF(K, L) = F(zK, zL)

10 % larger input of capital and labour raises output also by 10 %.

$$z = \frac{1}{L} \Rightarrow$$

$$\frac{Y}{L} = F(\frac{K}{L}, 1)$$

$\dfrac{Y}{L} = y =$ **output per capita**

$\dfrac{K}{L} = k =$ **capital intensity (capital stock per capita)**

$$y = F(k, 1) = f(k)$$

Output per capita is a function of capital intensity

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The Cobb-Douglas case

Suppose that $Y = K^{\alpha} L^{1-\alpha}$:

$$y = \frac{Y}{L} = \frac{K^{\alpha} L^{1-\alpha}}{L} = K^{\alpha} L^{-\alpha} = \left(\frac{K}{L}\right)^{\alpha} = k^{\alpha}$$

Including total factor productivity (A) so that $Y = AK^{\alpha} L^{1-\alpha}$:

$$y = \frac{Y}{L} = \frac{AK^{\alpha} L^{1-\alpha}}{L} = AK^{\alpha} L^{-\alpha} = A\left(\frac{K}{L}\right)^{\alpha} = Ak^{\alpha}$$

<u>The Solow model</u>

(1)	$y = c + i$	Goods market equilibrium
(2)	$c = (1\text{-}s)\, y$	Consumption function, s is the savings rate
(3)	$y = f(k)$	Production function
(4)	$d = \delta k$	Capital depreciation, δ is the rate of depreciation
(5)	$\Delta k = i - \delta k$	Change in the capital stock

Change in the capital stock = Gross investment – Depreciation

The Solow model (cont.)

Substituting the consumption function (2) into the goods market equilibrium condition (1) gives:

$$y = (1\text{-}s)y + i$$

$$i = sy$$

Investment = Saving

Substitution of the production function into the investment-savings equality gives:

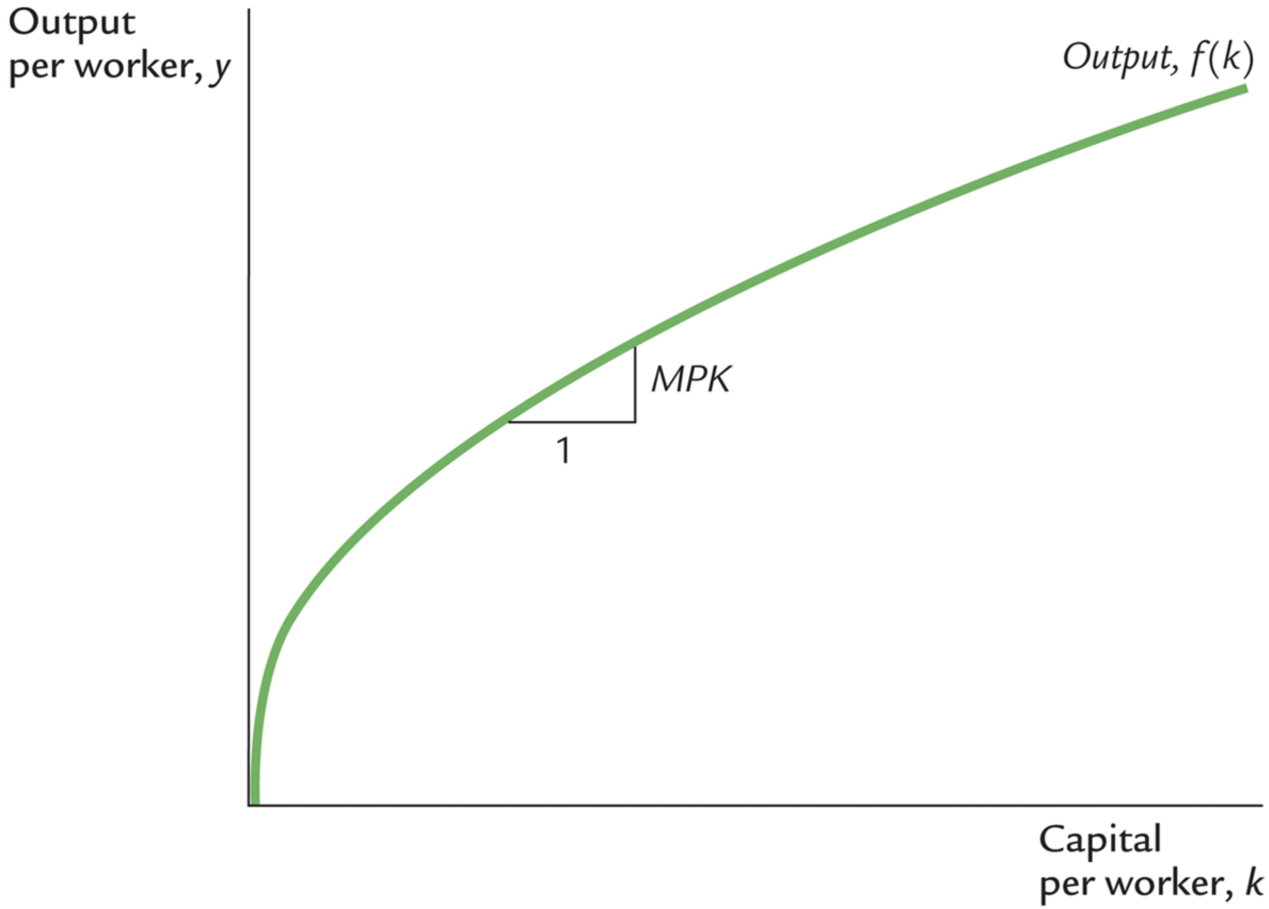
$$i = sf(k)$$

$$\because \quad \Delta\mathrm{k} = i - \delta k \ = sf(k) - \delta k$$

In a steady state, the capital stock is unchanged from period to period, i.e. $\Delta k = 0$ and thus:

$$sf(k) = \delta k$$

Figure 7.1 The production function



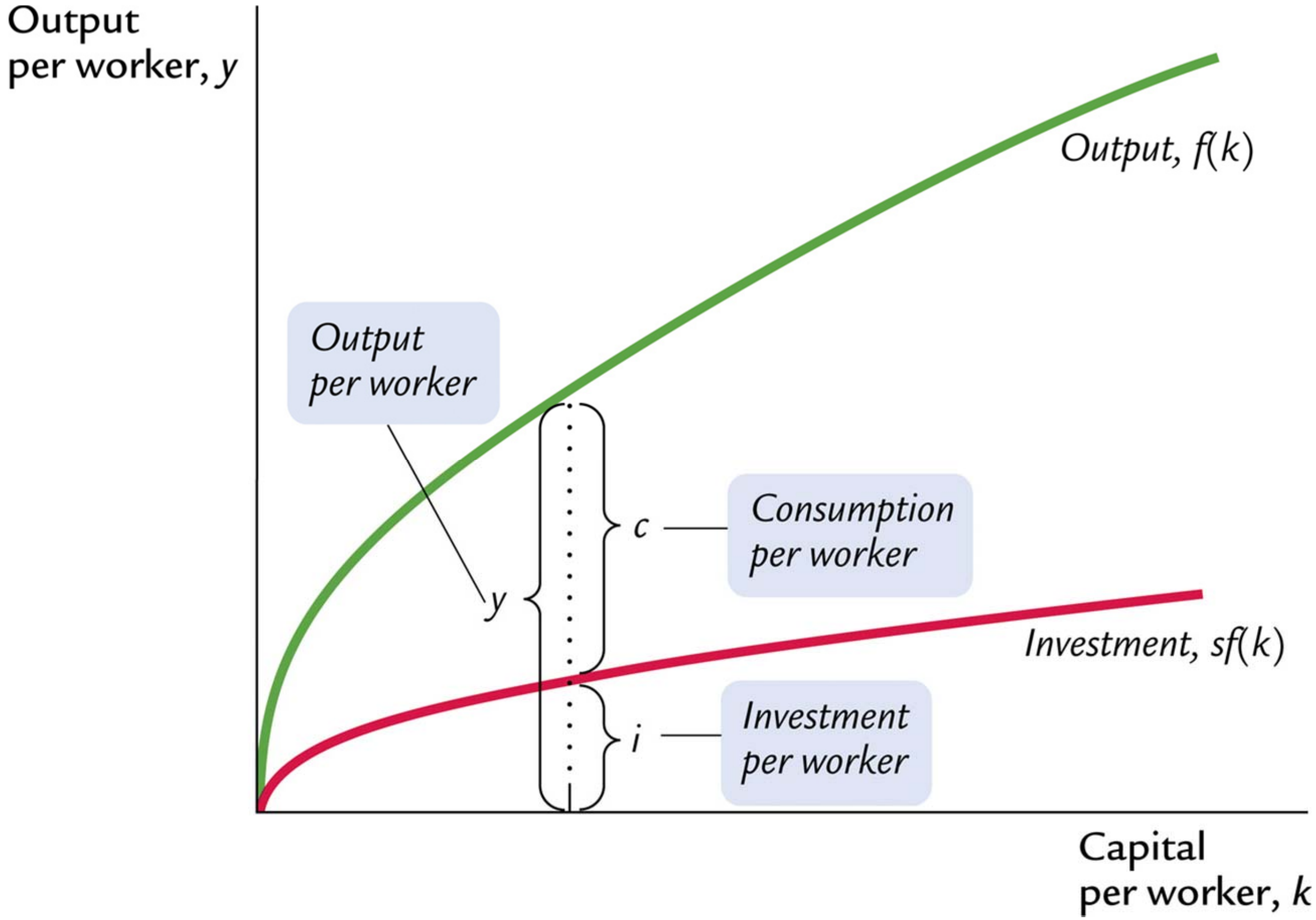


Figure 7-2: Output, consumption and investment

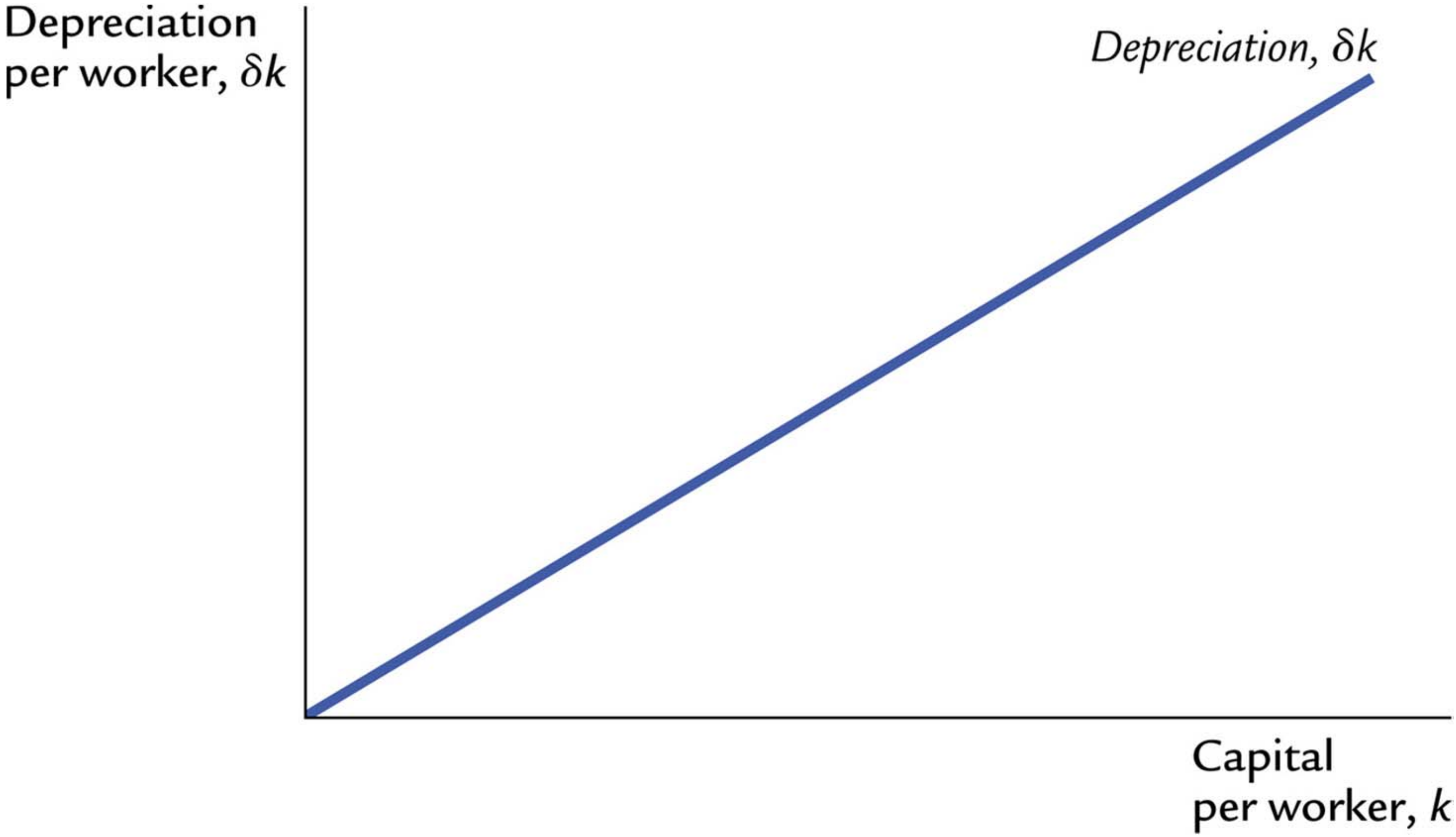


Figure 7-3: Depreciation

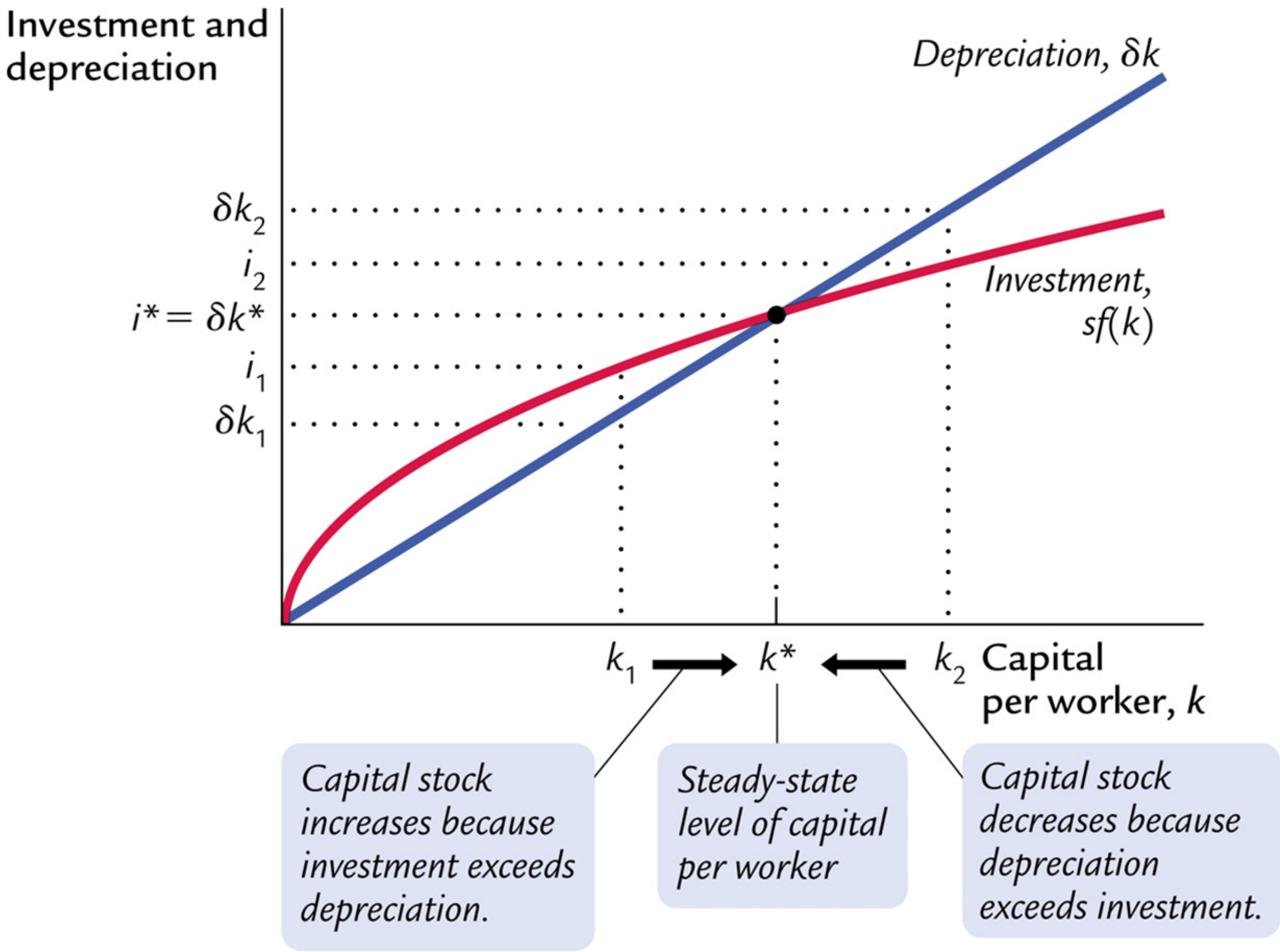


Figure 7-4: Investment, depreciation and the steady state

<u>Convergence of GDP per capita</u>

- Countries with different initial GDP per capita will converge (if they have the same production function, the same savings rate and the same depreciation rate).
- The catch-up factor
- Strong empirical support for the hypothesis that GDP growth is higher the lower is initial GDP per capita

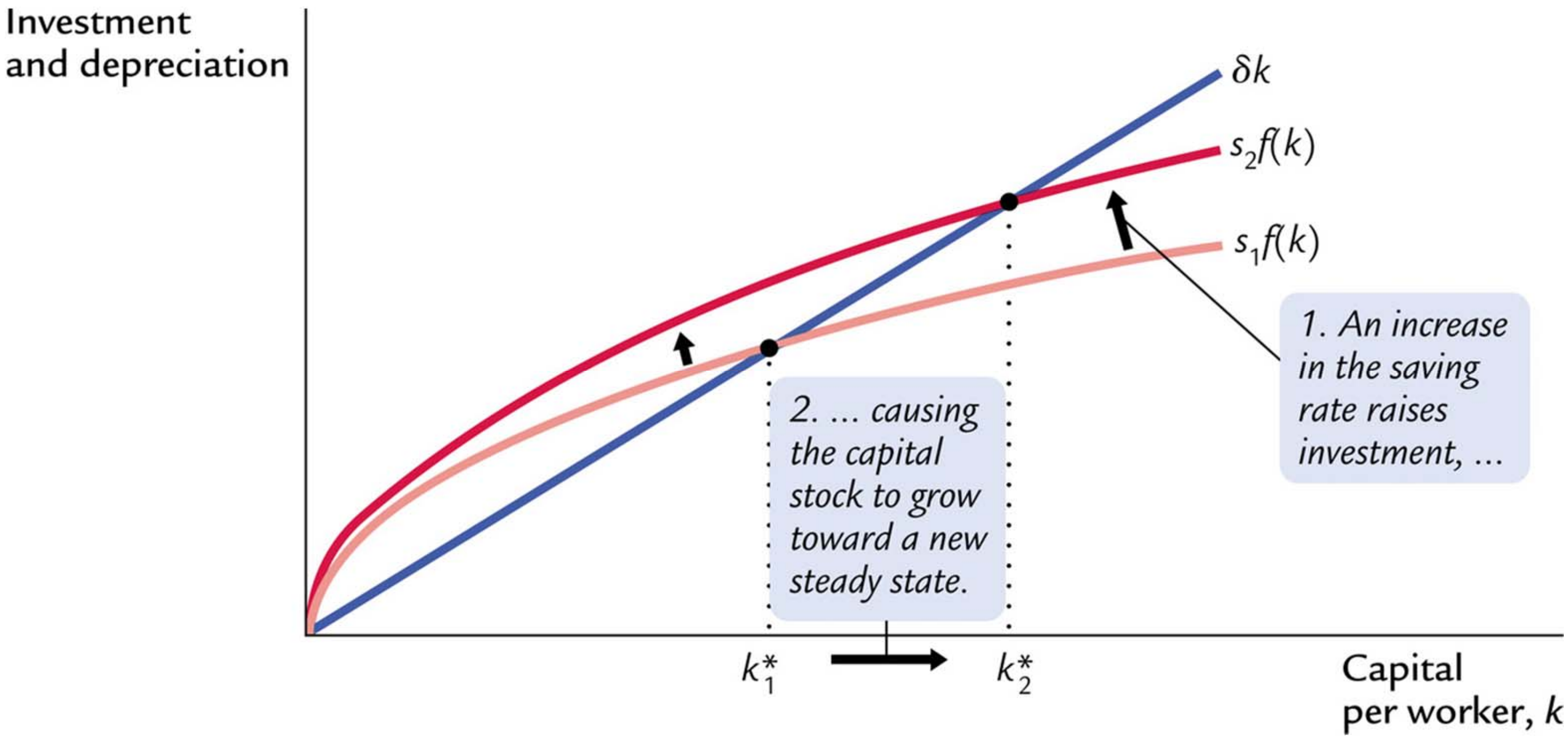


Figure 7-5: An increase in the saving rate

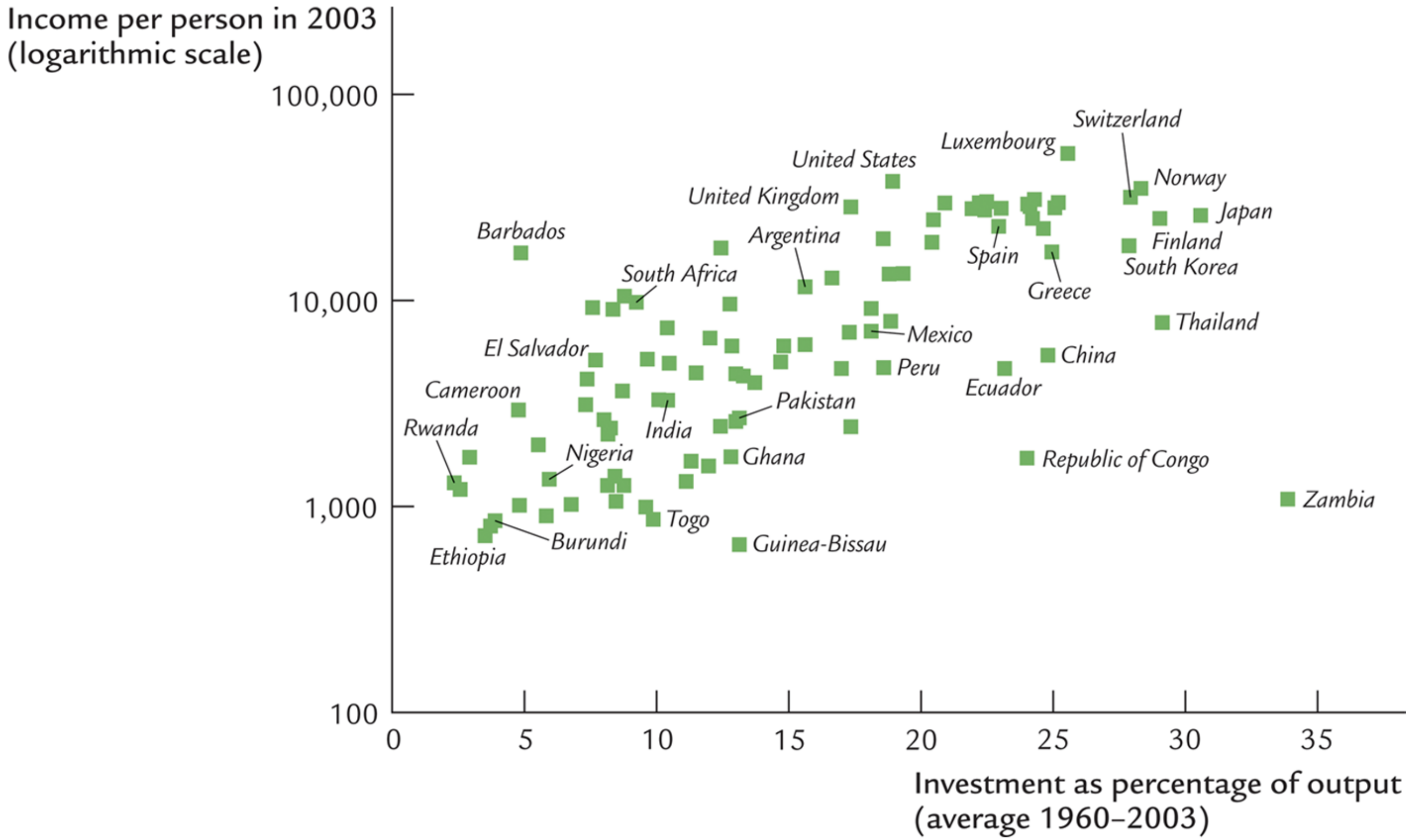


Figure 7-6: International evidence on investment rates and income per person

Golden rule of capital accumulation

Which savings rate gives the highest per capita consumption in the steady state?

$\boldsymbol{y = c + i}$

$\boldsymbol{c = y - i}$

In a steady state, gross investment equals depreciation:

$\boldsymbol{i = \delta k}$

Hence:

$\boldsymbol{c = f(k) - \delta k}$

Consumption is maximised when the marginal product of capital equals the rate of depreciation, i.e. ***MPC*** **=** $\boldsymbol{\delta}$

Mathematical derivation

The first-order condition for maximisation of the consumption function:

$$\partial c / \partial k = f_k - \delta = 0$$

$$f_k = \delta$$

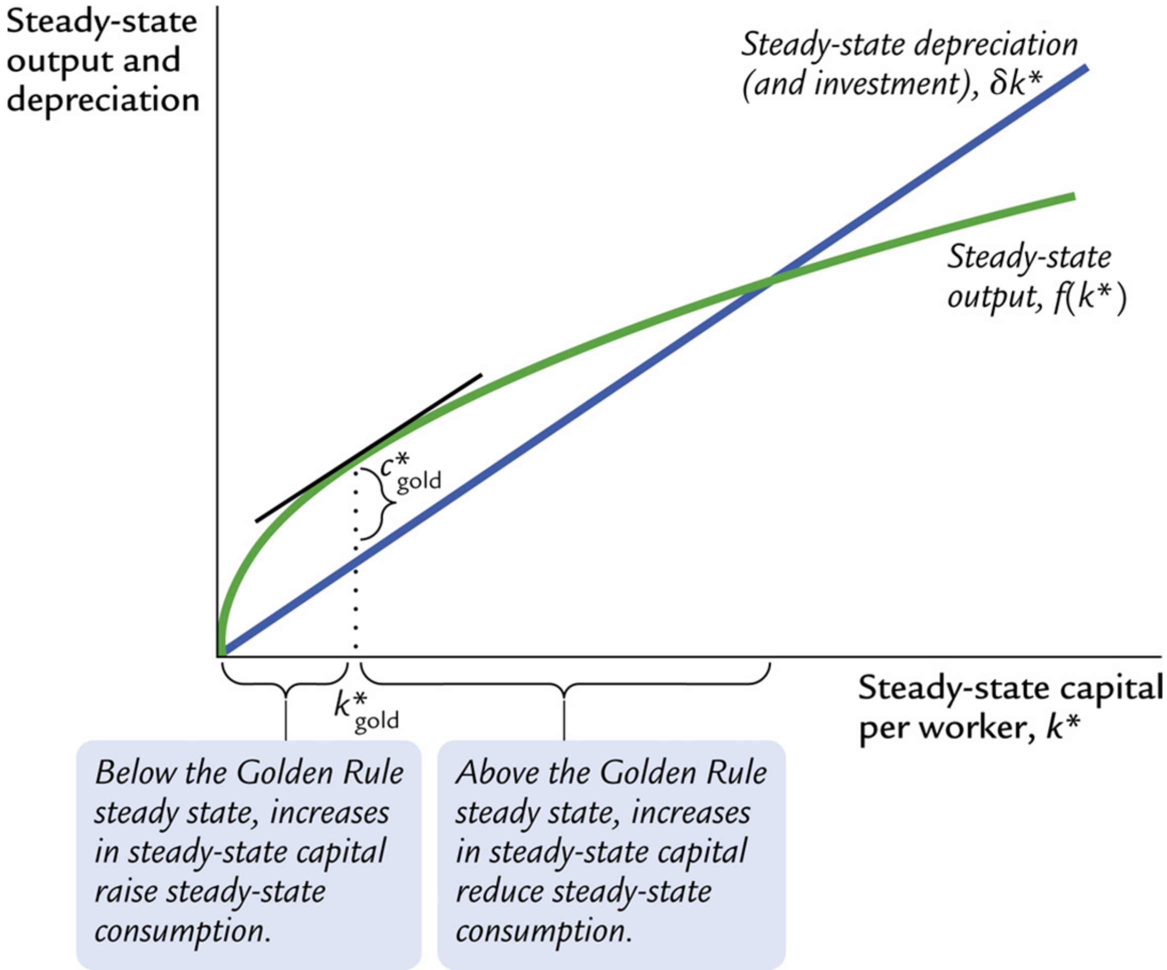


Figure 7-7: Steady-state consumption

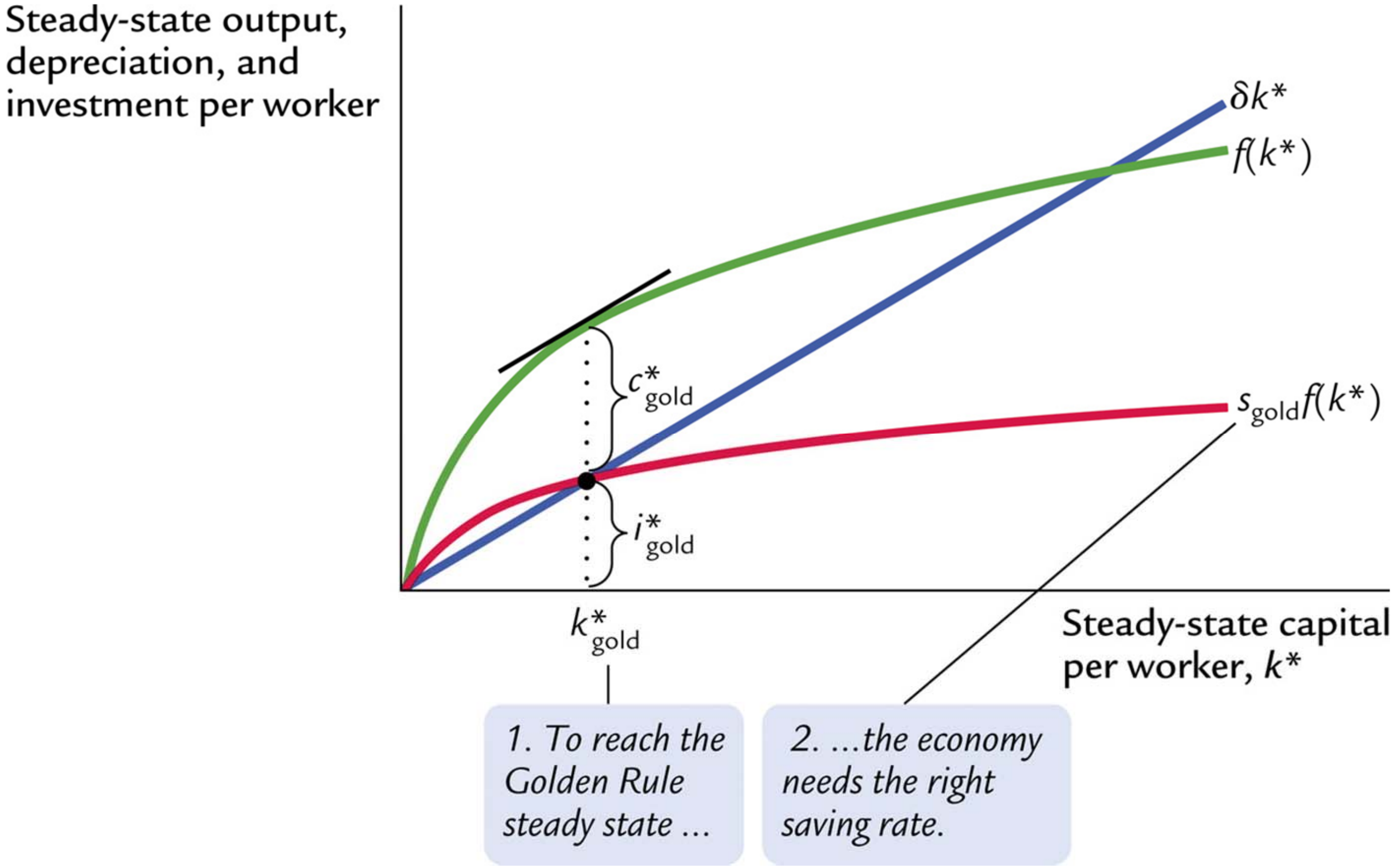


Figure 7-8: The saving rate and the golden rule

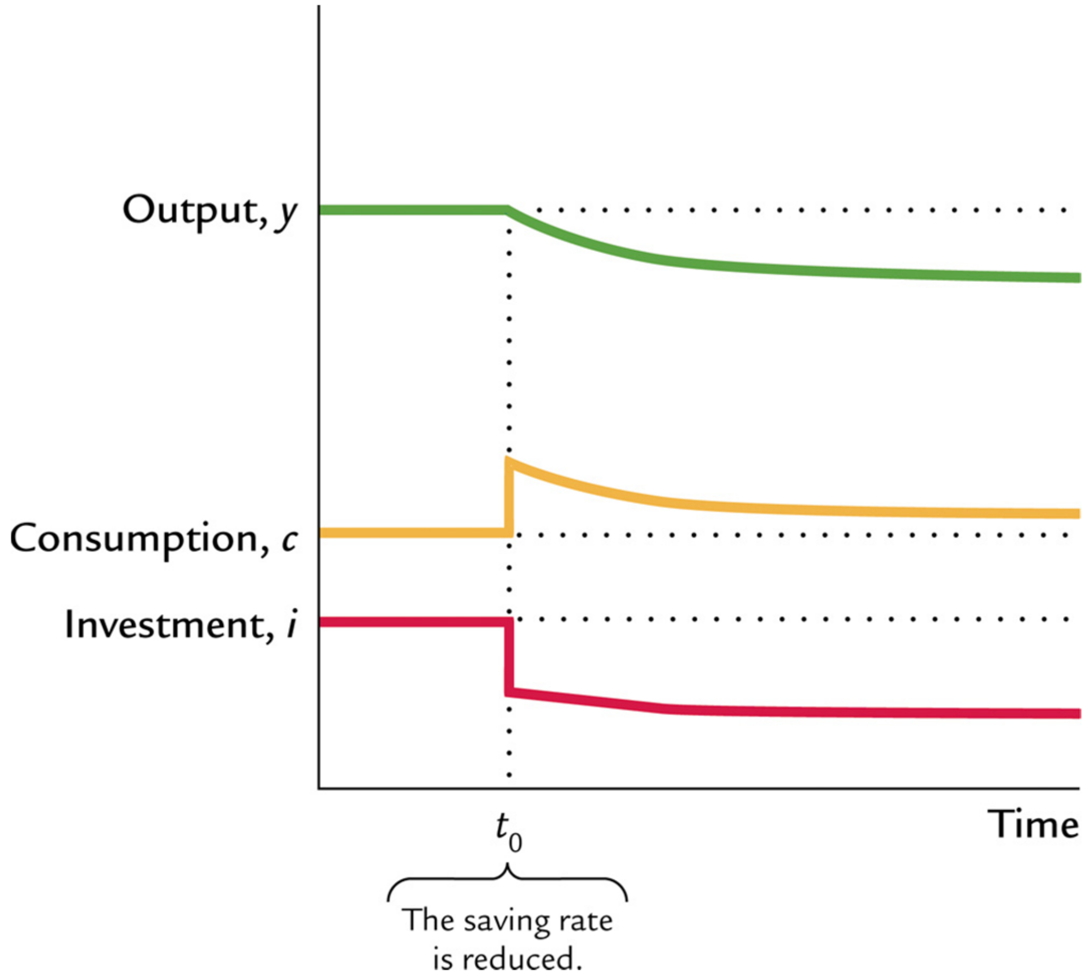


Figure 7-9: Reducing saving when starting with more capital than in the golden rule steady state

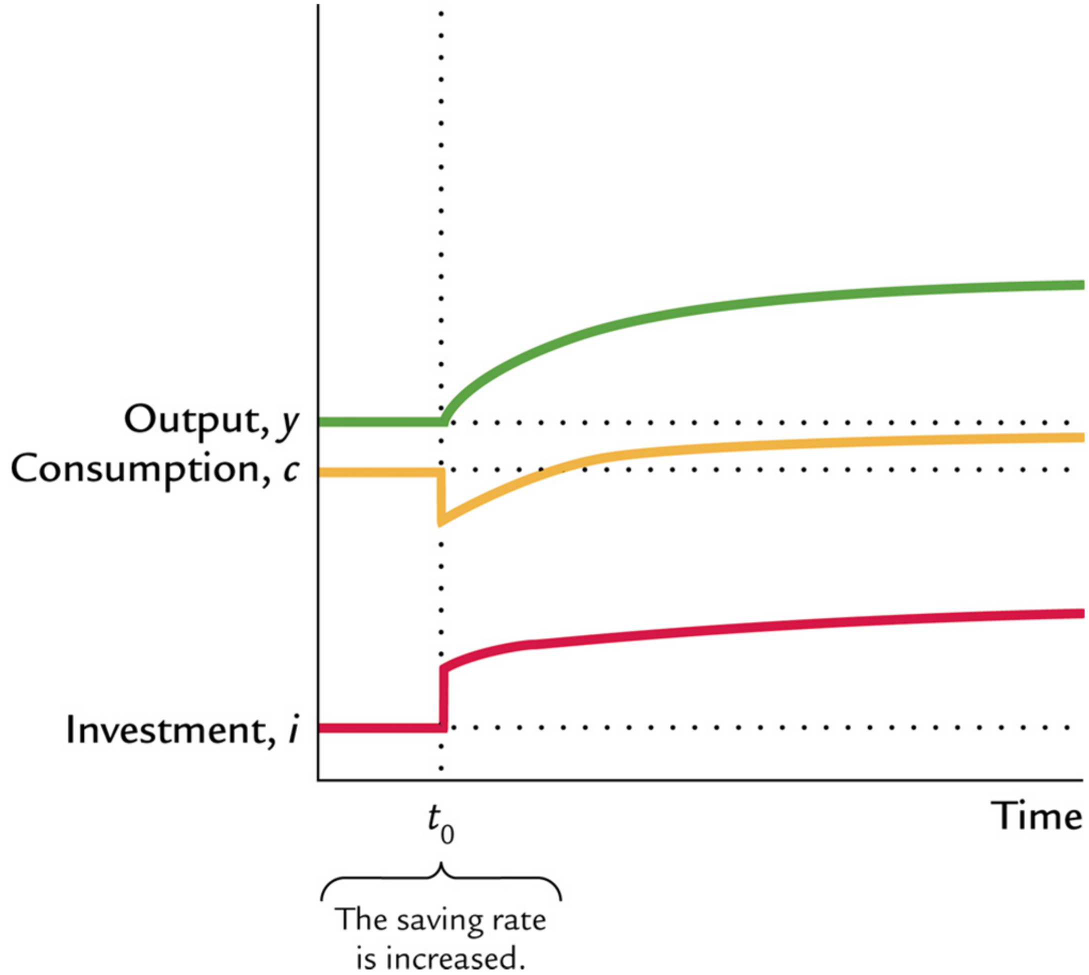


Figure 7-10: Increasing saving when starting with less capital than in the golden rule steady state

A steady state with population growth

$$n = \frac{\Delta L}{L} = \text{population growth}$$

$$\Delta k = i - \delta k - nk$$

Change in capital intensity ($k = K/L$) = Gross investment – Depreciation – Reduction in capital intensity due to population growth

In a steady state:

$$\Delta k = i - \delta k - nk = 0, \text{ i.e. } i = (\delta + n)k = 0$$

Derivation of the capital growth equation

K = capital stock, I = gross investment, L = population
$k = K/L$ = capital stock per worker (capital intensity)
$i = I/L$ = gross investment per worker

$$\Delta K = I - \delta K$$

$$\frac{\Delta K}{K} = \frac{I}{K} - \delta$$

Use that:

$$\frac{\Delta k}{k} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} \text{ and } \frac{\Delta L}{L} = n$$

$$\frac{\Delta k}{k} \approx \frac{I}{K} - \delta - n$$

Hence:

$$\frac{\Delta k}{k} \approx \frac{I}{L} \bullet \frac{L}{K} - \delta - n$$

$$\frac{\Delta k}{k} \approx \frac{i}{k} - \delta - n$$

Multiplying by k gives:

$$\Delta k \approx i - \delta k - nk = i - (\delta + n)k$$

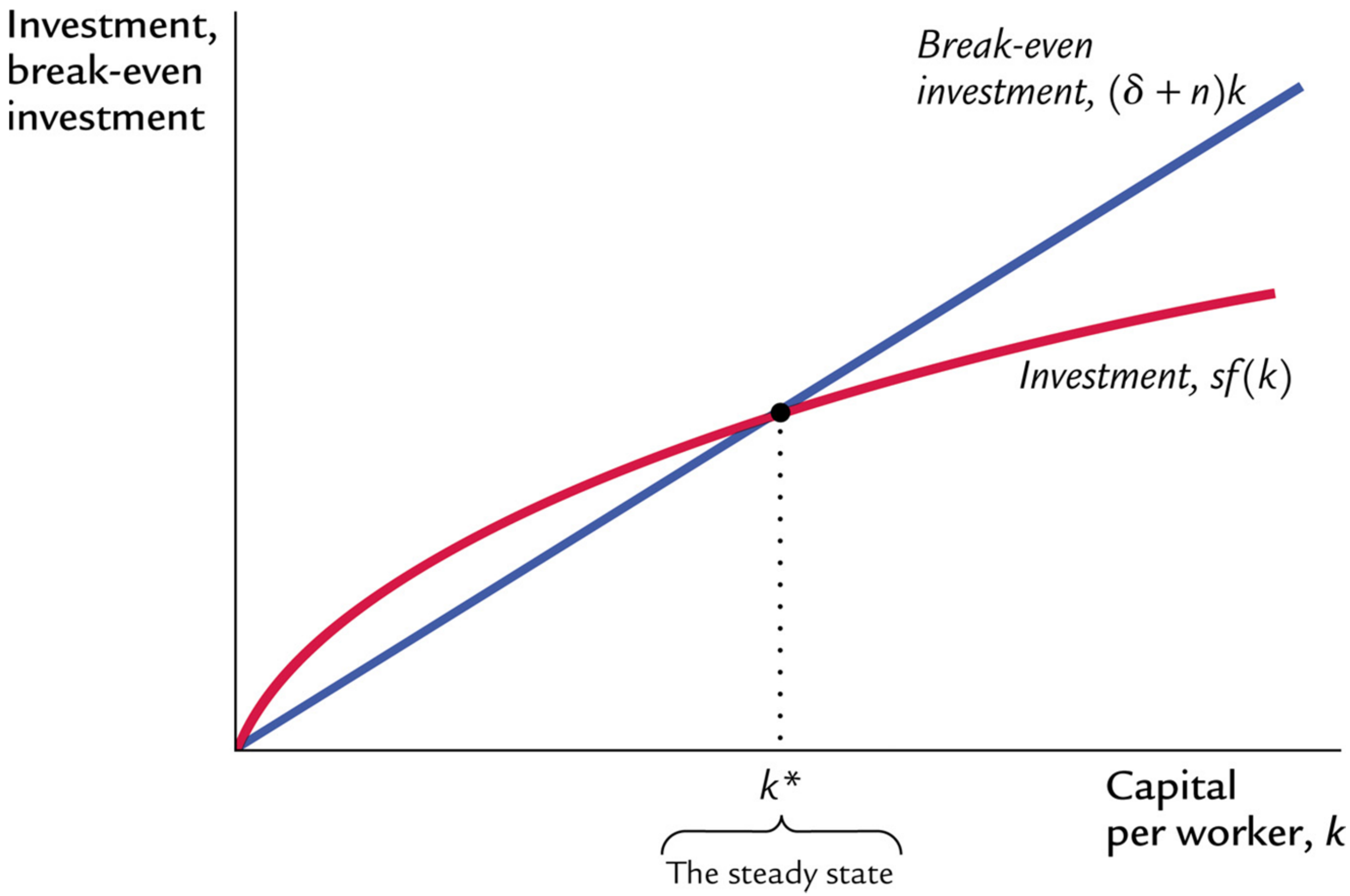


Figure 7-11: Population growth in the Solow model

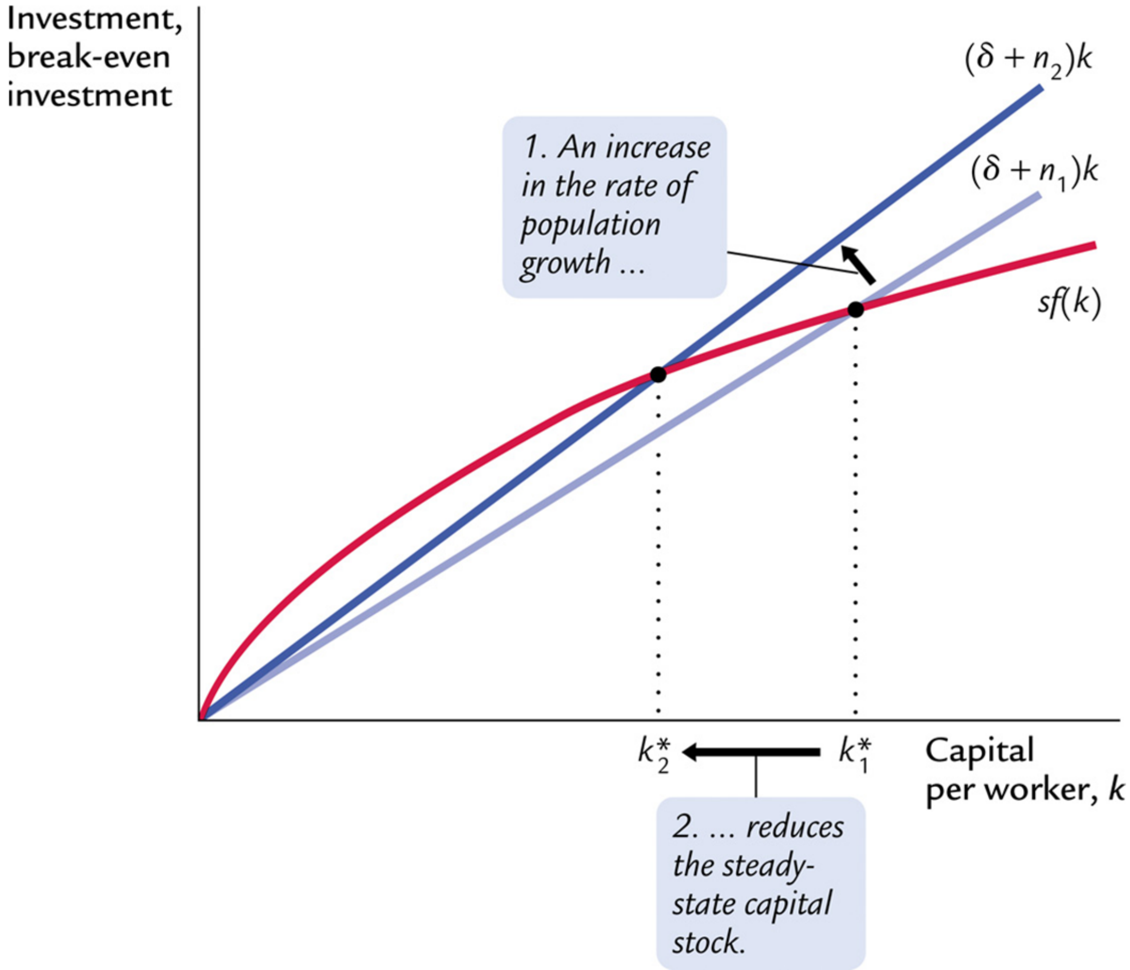


Figure 7-12: The impact of population growth

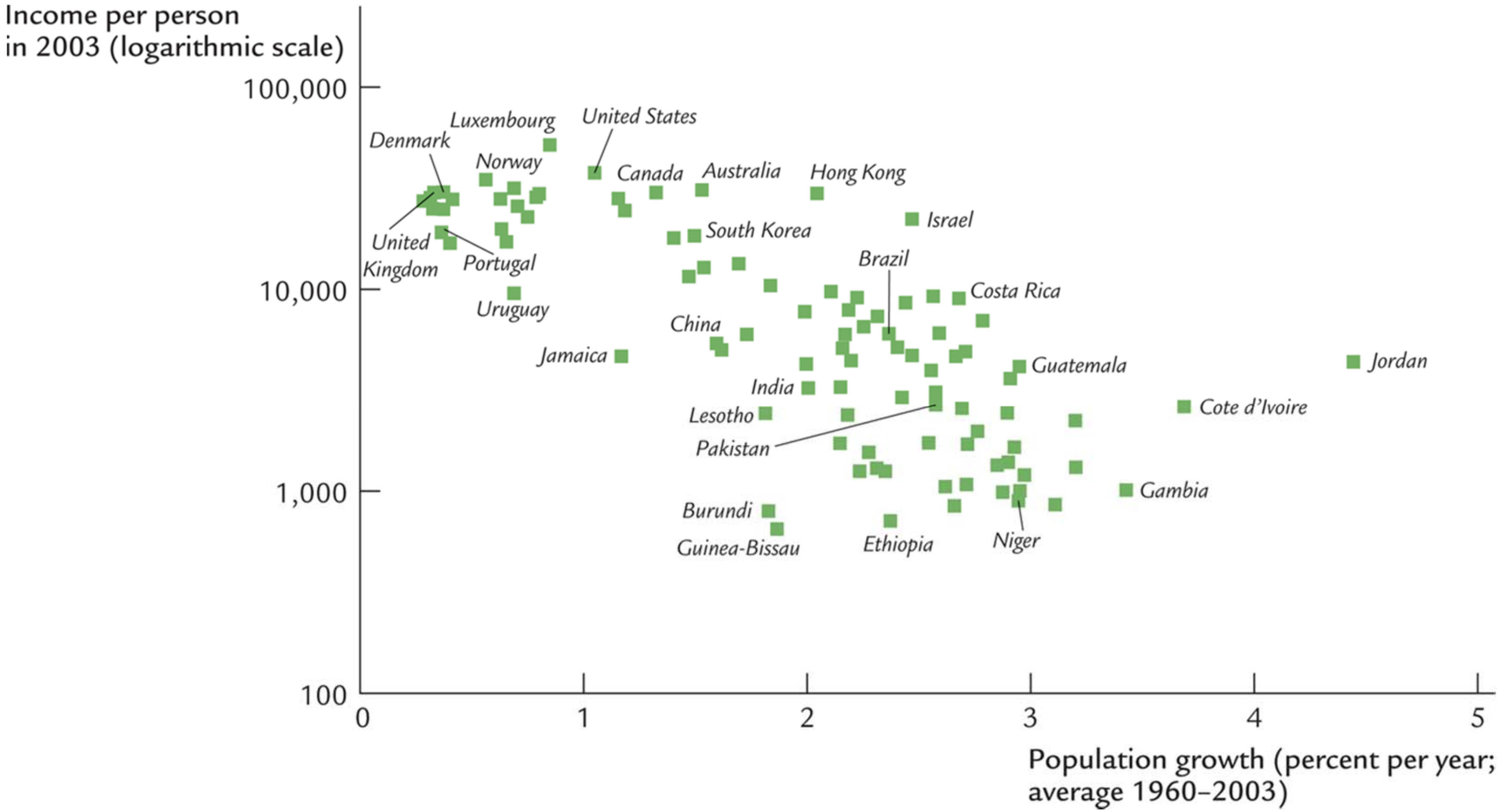


Figure 7-13: International evidence on population growth and income per person

A steady state with population growth

$$Y = F(K, L)$$

$$\frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

In a steady state, $k = K/L$ is constant. Because

$$\frac{\Delta k}{k} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} = 0,$$

We have

$$\frac{\Delta K}{K} = \frac{\Delta L}{L} = n$$

$$\because \text{ är } \frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} = \alpha n + (1-\alpha) n = n$$

GDP growth = Population growth

Golden rule with population growth

c = y – i = f(k) – (δ + n)k

Consumption per capita is maximised if *MPC = δ + n,* i.e. if the marginal product of capital equals the sume of the depreciation rate and population growth

Alternative formulation: **The net marginal product of capital after depreciation (*MPK – δ*) should equal population growth (*n*)**

Mathematical derivation

Differentiation of c-function w.r.t *k* gives:

$$\partial c / \partial k = f_k - (\delta + n) = 0$$

$$f_k = \delta + n$$

<u>Alternative perspectives on population growth</u>

1. **Malthus (1766-1834)**
    - **population will grow up to the point that there is just subsistence**
    - **man will always remain in poverty**
    - **futile to fight poverty**

2. **Kremer**
    - **population growth is a key driver of technological growth**
    - **faster growth in a more populated world**
    - **the most successful parts of the world around 1500 was the old world (followed by Aztec and Mayan civilisations in the Americas; hunter-gatherers of Australia)**

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Labour-augmenting technical progress

$$Y = F(K, L \bullet E)$$

E = labour efficiency

$L \bullet E$ = efficiency units of labour

$$y = \frac{Y}{LE} = F(\frac{K}{LE}, 1) = F(k, 1) = f(k)$$

$$k = \frac{K}{LE}$$

Steady state

L grows by n % per year

E grows by g % per year

$\Delta k = sf(k) - (\delta + n + g)k = 0$

Gross investment = Depreciation + Reduction in capital intensity because of population growth + Reduction in capital intensity because of technological progress

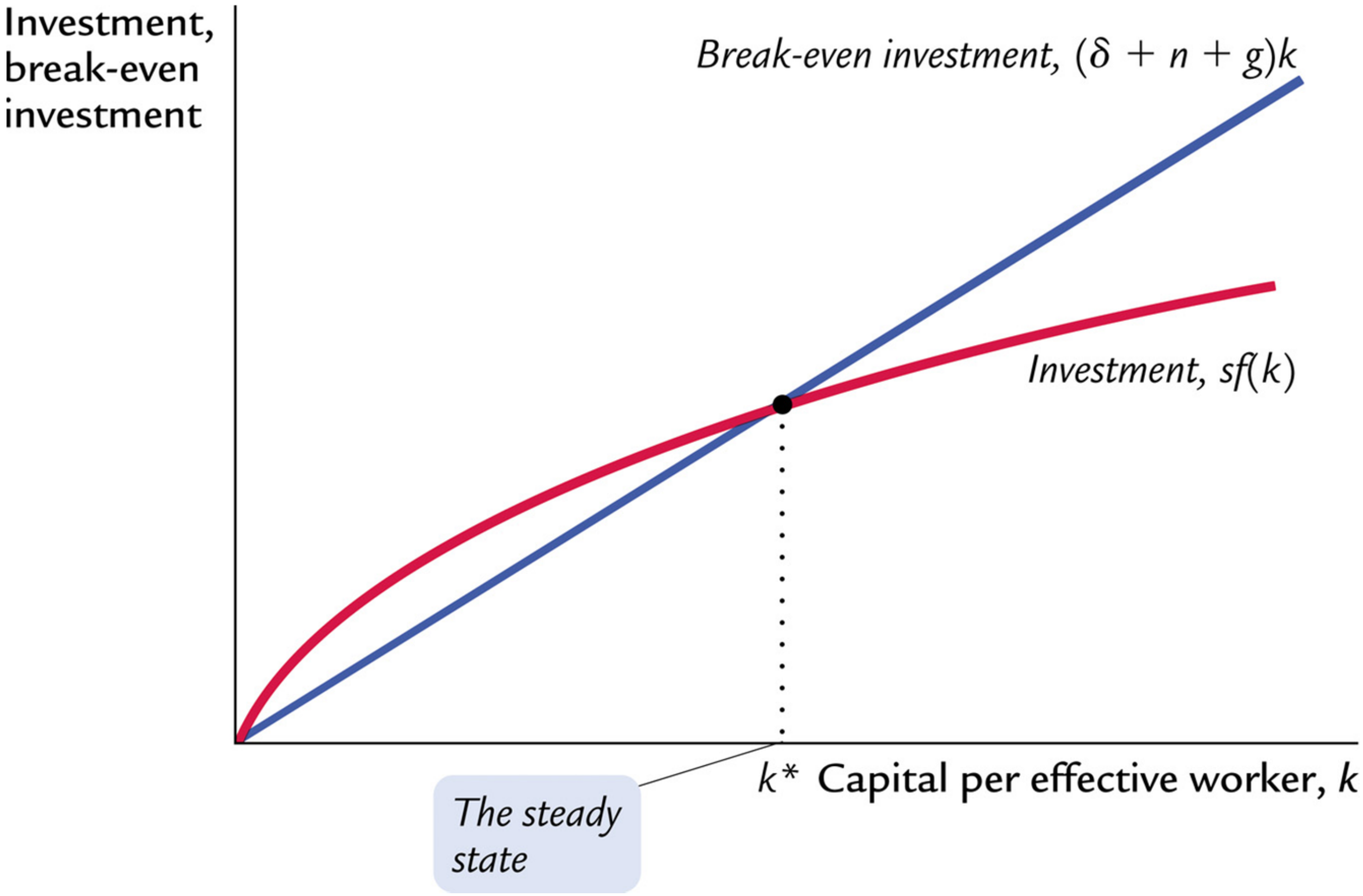

Figure 8-1: Technological progress and the Solow growth model

<u>Growth and labour-augmenting technological progress</u>

$$Y = K^{\alpha}(LE)^{1-\alpha}$$

$$\frac{\Delta Y}{Y} \approx \alpha\frac{\Delta K}{K} + (1-\alpha)(\frac{\Delta L}{L} + \frac{\Delta E}{E})$$

In a steady state *K/LE* is constant

$$(\Delta L/L + \Delta E/E) = n + g \Rightarrow \Delta K/K = n + g.$$

$$\frac{\Delta Y}{Y} \approx \alpha(n + g) + (1-\alpha)(n + g) = n + g$$

GDP growth = population growth+ technological progress

$$\frac{\Delta y}{y} \approx \frac{\Delta Y}{Y} - \frac{\Delta L}{L} = n + g - n = g$$

Growth in GDP per capita = rate of technological progress

<u>Table 8-1</u>: Steady-State growth rates in the Solow model with technological progress

Variable	**Symbol**	**Steady-state growth rate**
Capital per effective worker	$\boldsymbol{k} = \boldsymbol{K}/(\boldsymbol{L} \times \boldsymbol{E})$	0
Output per effective worker	$\boldsymbol{y} = \boldsymbol{Y}/(\boldsymbol{L} \times \boldsymbol{E})$	0
Output per worker	$(\boldsymbol{Y}/\boldsymbol{L}) = \boldsymbol{y} \times \boldsymbol{E}$	$\boldsymbol{g}$
Total output	$\boldsymbol{Y} = \boldsymbol{y} \times \boldsymbol{E} \times \boldsymbol{L}$	$\boldsymbol{n} + \boldsymbol{g}$

<u>Golden rule with technological progress</u>

$c = f(k) - (\delta + n + g)k$

Consumption per efficiency unit is maximised if $MPK = \delta + n + g$

The marginal product of capital should equal the sum of depreciation, population growth and technological progress

<u>Alternative formulation:</u> The net marginal product (*MPK* - δ) should equal GDP growth (*n* + *g*).

<u>Mathematical derivation</u>

Differentiation w.r.t. *k*:

$$\partial c / \partial k = f_k - (\delta + n + g) = 0$$

$$f_k = \delta + n + g$$

Real world capital stocks are smaller than according to the golden rule. The current generation attaches a larger weight to its own welfare than according to the golden rule.

Endogenous or exogenous growth

- **In the Solow model growth is exogenously determined by population growth and technological progress**
- **Recent research has focused on the role of human capital**
- **A higher savings rate or investment in human capital do not change the rate of growth in the steady state**
- **The explanation is decreasing marginal return of capital (*MPK* is decreasing in *K*)**

The AK-model

$Y = AK$

$\Delta K = sY - \delta K$

Assume *A* to be fixed!

$\Delta Y/Y = \Delta K/K$

$\Delta K/K = sAK/K – \delta K/K = sA - \delta$

$\Delta Y/Y = sA - \delta$

- **A higher savings rate *s* implies permanently higher growth**
- **Explanation: constant returns to scale for capital**
- **Complementarity between human and real capital**

A two-sector growth model

- **Business sector**
- **Education sector**

$Y = F[K, (1\text{-}u)EL]$ **Production function in business sector**

$\Delta E = g(u)E$ **Production function in education sector**

$\Delta K = sY - \delta K$ **Capital accumulation**

u **= share of population in education**

$\Delta E/E = g(u)$

- **A higher share of population, *u*, in education raises the growth rate permanently (cf *AK*-model – here human capital)**
- **A higher savings rate, *s*, raises growth only temporarily as in the Solow model**

Human capital in growth models

1. **Broad-based accumulation of knowledge in the system of education**

2. **Generation of ideas and innovations in research-intensive R&D sector**
3. **Learning by doing at the work place**

Policy conclusions

1. **Basic education – incentives for efficiency in the education system – incentives to choose and complete education**

2. **Put resources in top-quality R&D**

3. **Life-long learning in working life**

Technological externalities / knowledge spillovers

Role of institutions

- **Quality of institutions determine the allocation of scarce resources**

- **Legal systems – secure property rights**
  - "helping hand" from government (Europe)
  - "grabbing hand" from government

- **Acemoglu / Johnson /Robinson**
  - European settlers in colonies preferred moderate climates (US, Canada, NZ)
  - European-style institutions
  - Earlier institutions strongly correlated with today's institutions

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Political Regimes and Growth

- Democracy versus Autocracy (Dictatorship)
- Fundamental difference between regimes: Autocratic leader can do what he wants as he does not rely on voters for re-election
- Democracy typically thought to promote growth (better institutions and policies, property rights, higher income equality)
- However: in democracies, special interest groups may try to block new technology and therefore retard development
- A benevolent (growth-promoting) autocrat may then fare better than a democratic leader
- Many of the growth miracle countries (East Asia) were dictatorships when their growth miracles took off (Larsson and Parente, 2010)

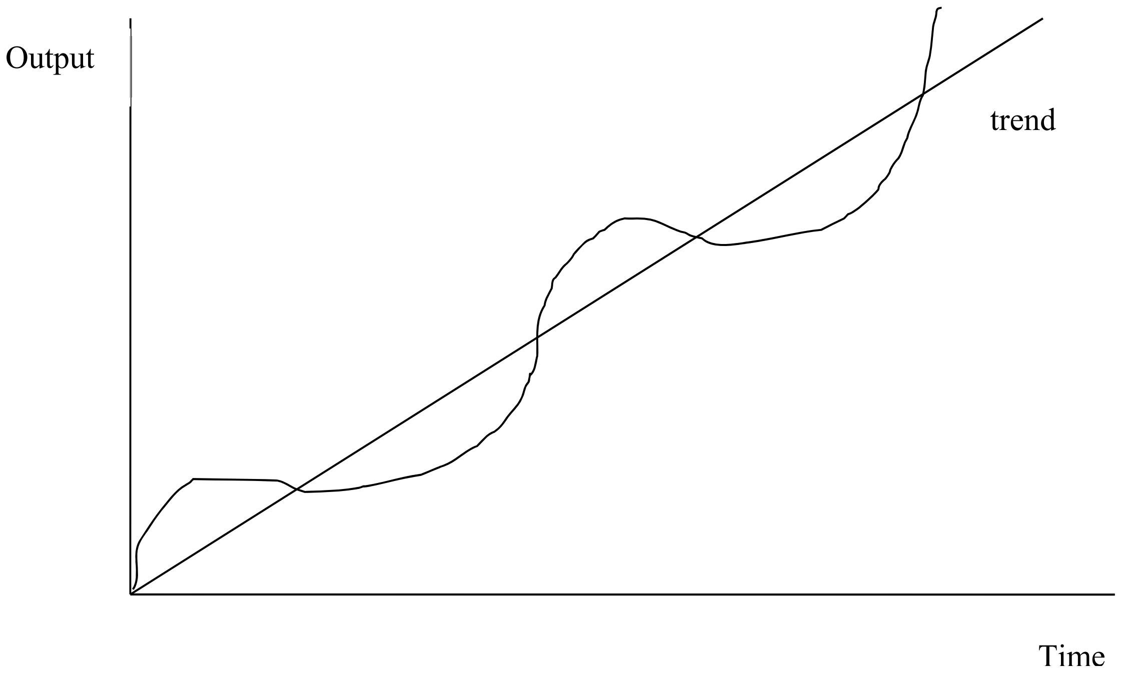
TABLE 8-2

Growth Around the World

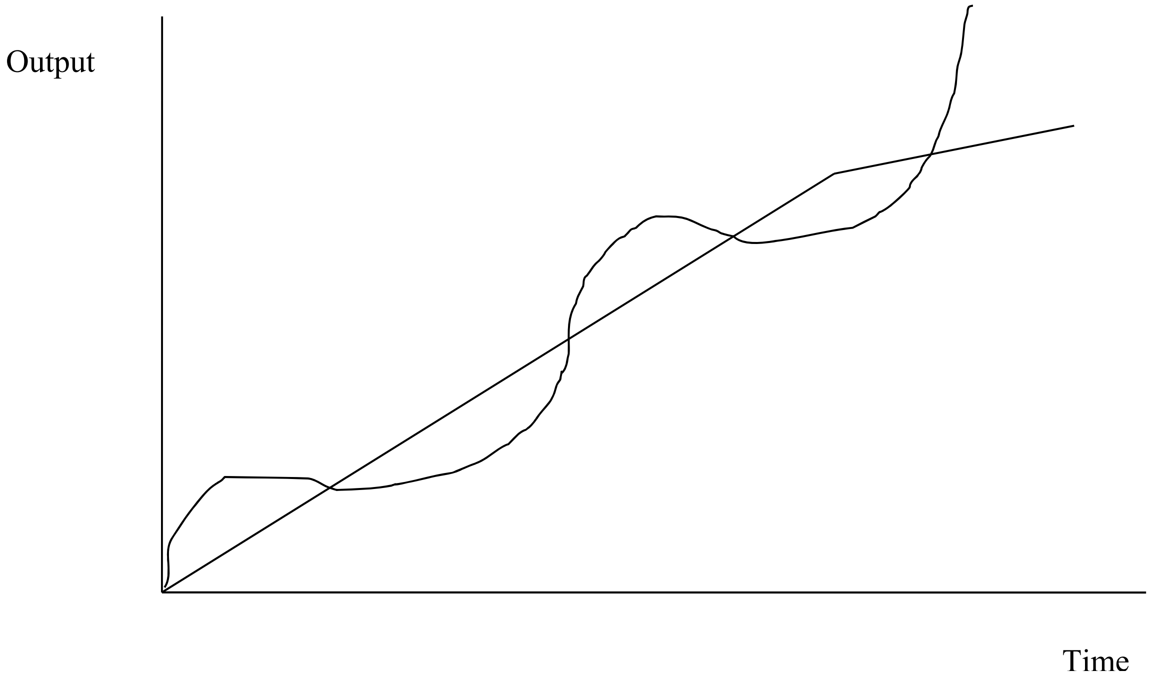
Country	GROWTH IN OUTPUT PER PERSON (PERCENT PER YEAR) 1948–1972	1972–1995	1995–2007
Canada	2.9	1.8	2.2
France	4.3	1.6	1.7
West Germany	5.7	2.0	
Germany			1.5
Italy	4.9	2.3	1.2
Japan	8.2	2.6	1.2
United Kingdom	2.4	1.8	2.6
United States	2.2	1.5	2.0

Source: Angus Maddison, *Phases of Capitalist Development* (Oxford: Oxford University Press, 1982); *OECD National Accounts;* and *World Bank: World Development Indicators.*

Temporary effects of a recession



Permanent effects of a recession



<u>Will the recession have long-run growth effects?</u>

- **Traditional view: a recession only represents a temporary reduction in resource utilisation**

- **Modern view a recession can have "permanent" effects on potential output growth**

<u>Effects on potential growth</u>

- **Slower growth of capital input**
  - **lower investment because of lower output and credit crunch in the short run and because of higher risk premia (higher interest rates and thus higher capital costs) in the medium run**
  - **capital becomes obstacle**

- **Higher structural unemployment**

- **Slower growth in total factor productivity**
  - **lower R&D expenditure**
  - **but also closing down of least efficient firms**

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