Lecture 8: Labour Economics and Wage-Setting Theory

Spring 2019

Lars Calmfors

Literature: Chapter 9 Cahuc-Carcillo-Zylberberg: 554-578 (recommended), 578-602

<u>Topics</u>

- **The Beveridge curve**
- **The competitive model with job reallocation**
- **The Mortensen-Pissarides matching model**
- **Matching function**
- **Labour demand**
- **Wage setting**
- **Social optimum in the matching model**

<u>Job re-allocation and matching</u>

- **Re-allocation in the labour market takes time.**
- **Simultaneous presence of vacancies and unemployed persons.**
- **Problems of matching give rise to frictional unemployment.**
- **Continuous process of job creation and job destruction.**
- **Probability for an unemployed to find a job depends on <u>labour market tightness</u> (number of vacant jobs per unemployed).**
- **Probability to fill a vacancy also depends (but negatively) on labour market tightness.**

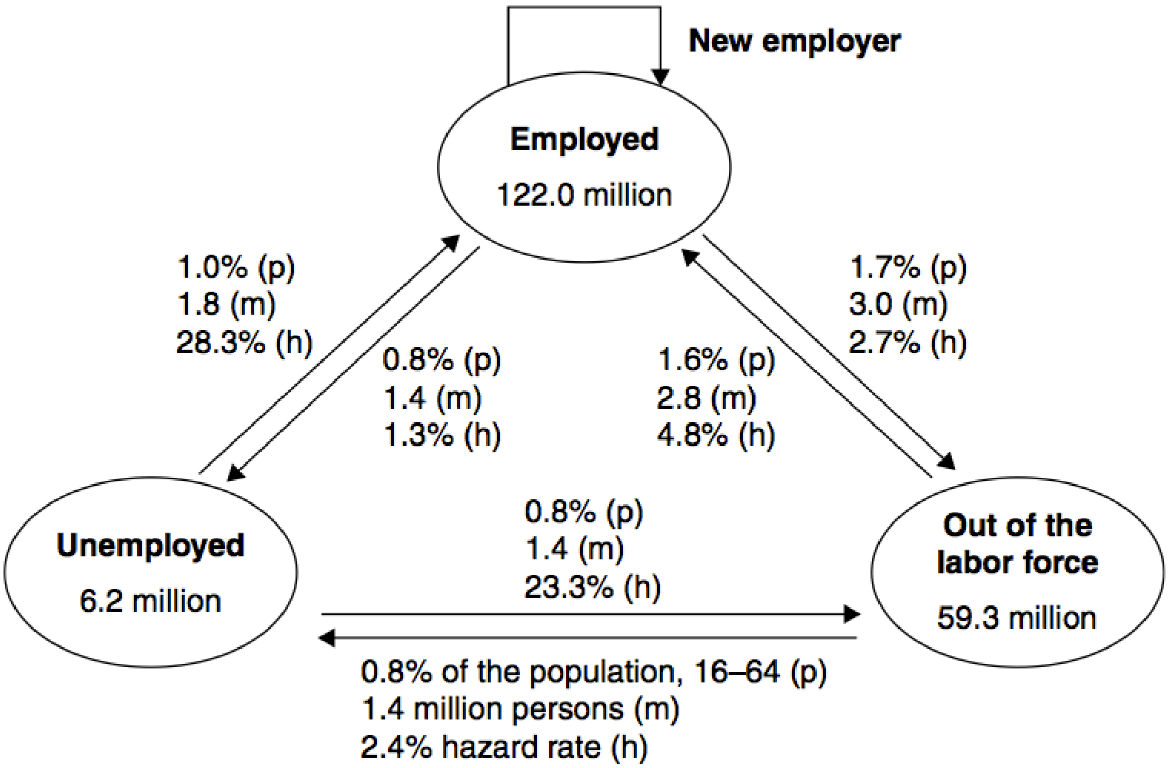


FIGURE 9.10
Average monthly worker flows in the United States. Current Population Survey, 1996–2003.

Source: Davis et al. (2006, figure 1).

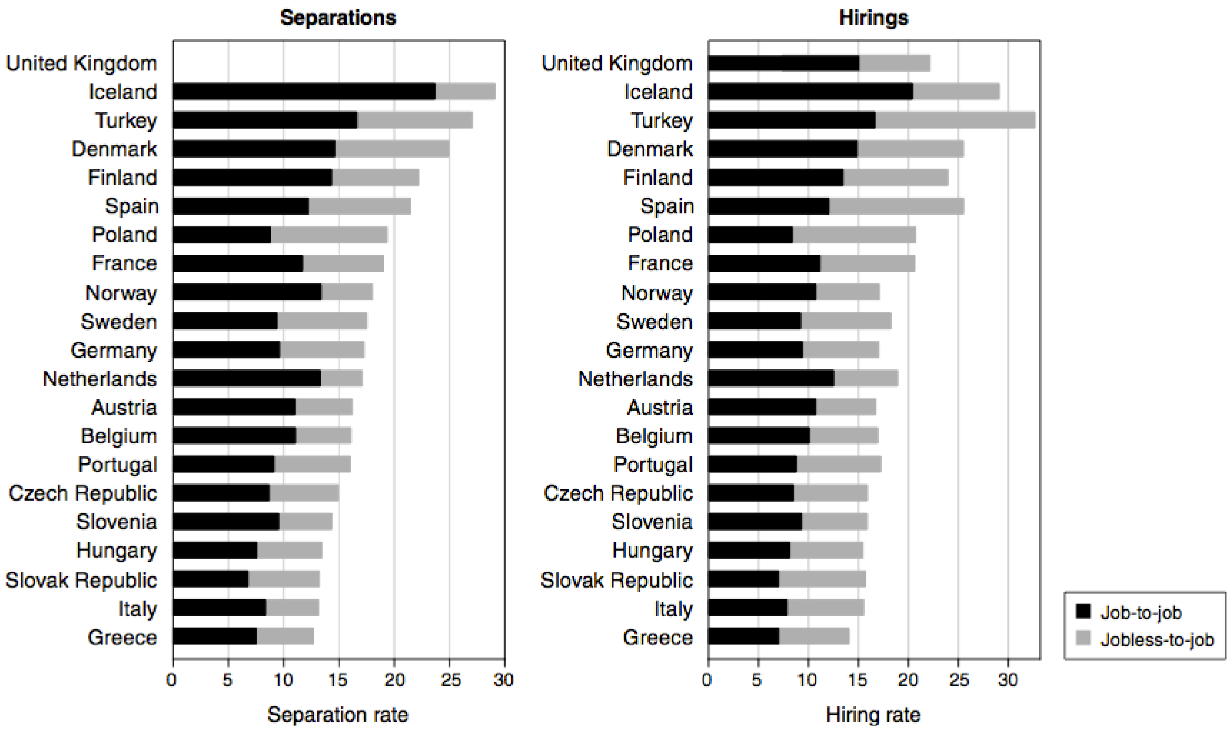


Figure 9.11
Job-to-job, jobless-to-job, and job-to-jobless flows in the European countries, 2000–2007.

Note: Country average rates expressed in percentages and adjusted for industry composition. Years around 2000–2007.

Source: OECD (2010, figure 3.2, p. 175).

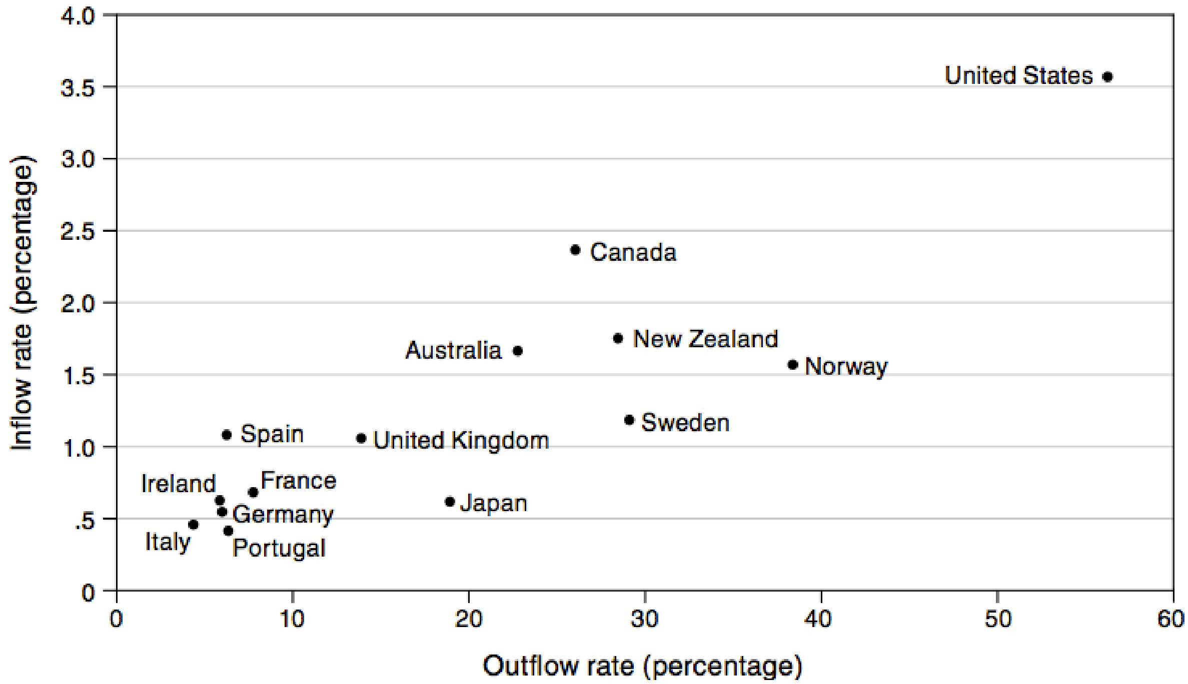


Figure 9.13
Unemployment inflow and outflow monthly rates in the OECD countries. The entry rate is the ratio between monthly entries into unemployment and the total number of employed persons during the month in question; the exit rate is the ratio between monthly exits from unemployment and the total number of unemployed persons during the month in question. The starting year for the available series varies between 1968 (for the United States) and 1986 (for New Zealand and Portugal). For all countries, the data end in 2009.

Source: Elsby et al. (2013).

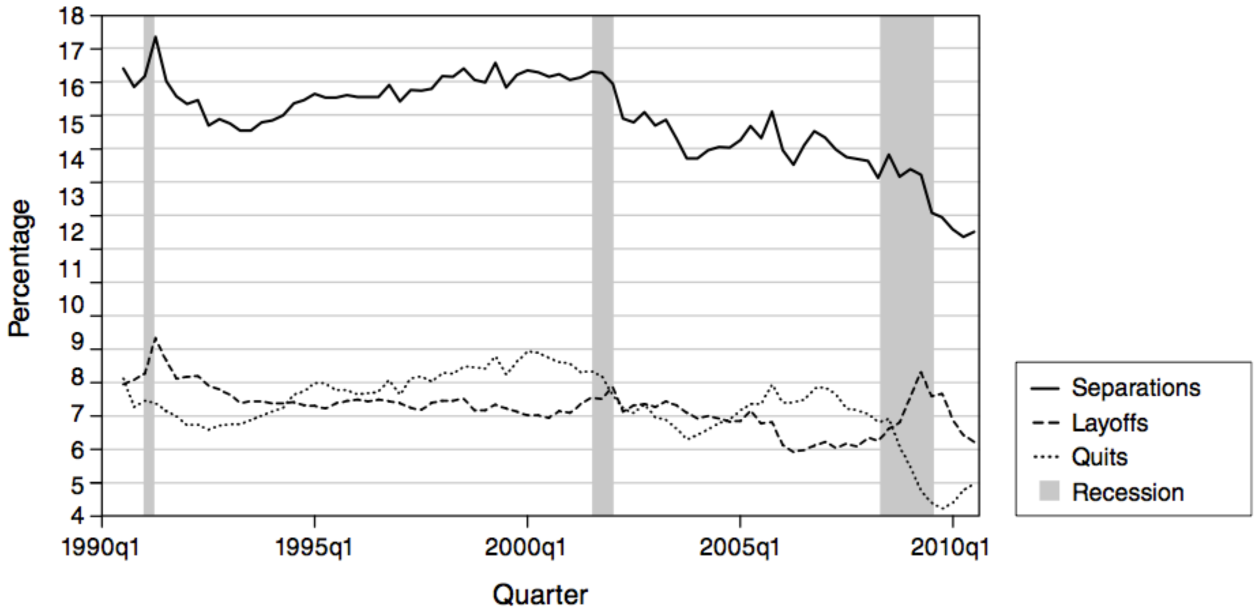


FIGURE 9.14
Quits, layoffs, and separations quarterly rates in the United States. Private sector, 1990q2–2010q2.

Source: Davis et al. (2012) database.

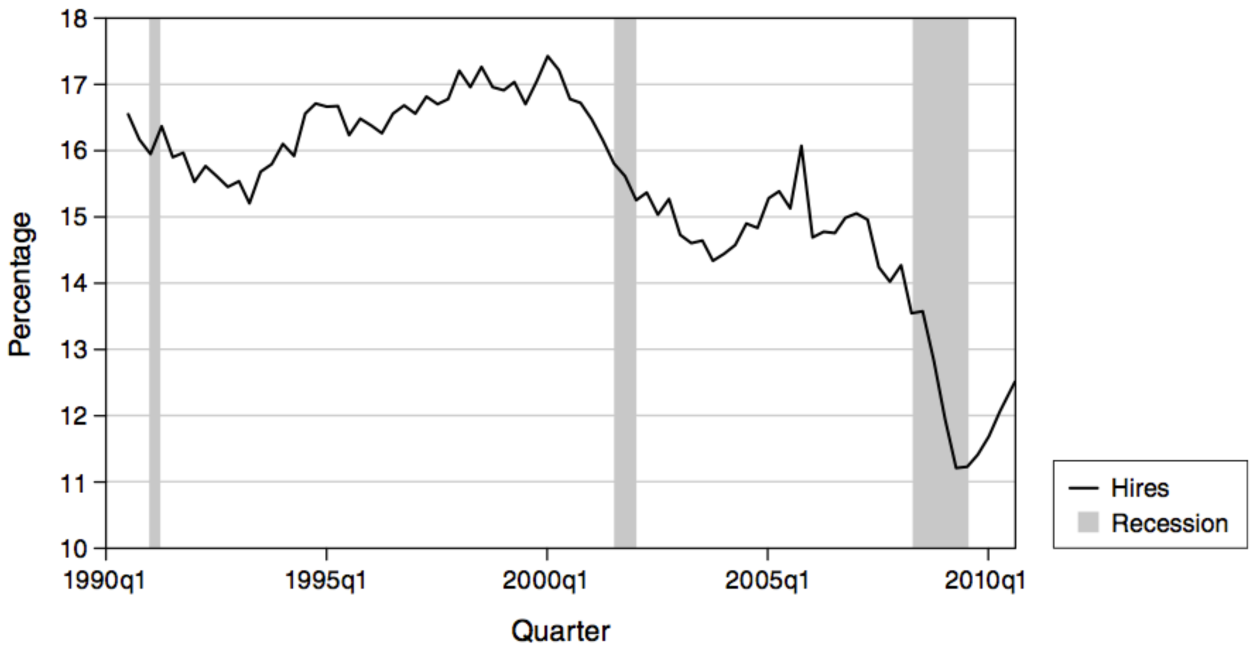


FIGURE 9.15
Hiring quarterly rates in the United States. Private sector, 1990q1–2010q2.

Source: Davis et al. (2012) database.

Unemployment dynamics

- Variation in unemployment depends both on variations in inflow and outflows
- In anglophone countries variation in outflows dominates
- More even split in European countries

The competitive model with job reallocation

- **The labour force consists of a large number of individuals with different reservation wages given by the cumulative distribution function** $H(.)$**.**
- **Labour supply is** $NH(w)$**.**
- **Firms face an adjustment cost** $C(\Lambda)$ **when hiring new workers, where** (Λ) **is turnover of workers**
  $C' > 0$

  $C'' > 0$ (convex adjustment cost)
- **Each worker can produce** y **goods.**
- L **= employment level.**
- **An exogenous proportion of jobs,** q**, is destroyed at each instant.**

$\pi = Ly - \left[wL + C(qL)\right]$ **in a steady state.**

Profit maximisation gives:

$$\frac{\partial \pi}{\partial L} = y - \left[w + qC'(qL)\right] = 0$$

$$y = qC'(qL) + w \qquad (1)$$

Marginal productivity = marginal adjustment cost of a job.

(1) defines labour demand.

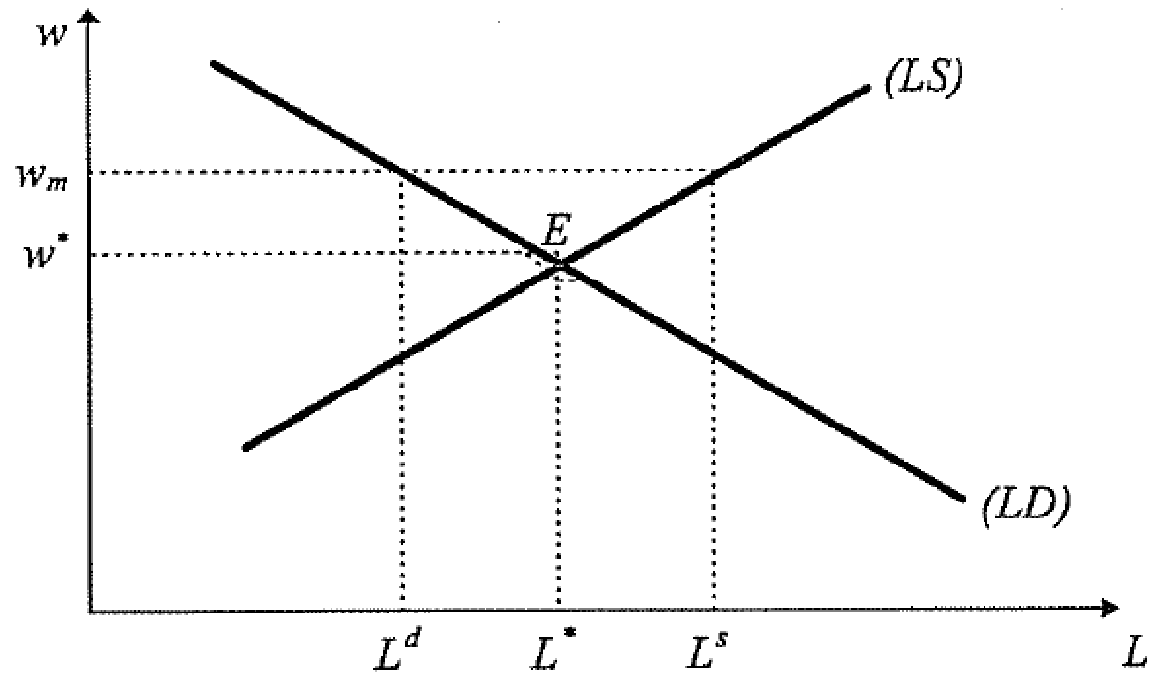


FIGURE 9.3
The competitive equilibrium.

- **An increase in the job destruction rate *q* increases the marginal adjustment cost and hence reduces labour demand (at a given wage).**

<u>Competitive equilibrium</u>

$$L^* = NH(w^*)$$

$$y = qC'\left[qNH(w^*)\right] + w^* \qquad (2)$$

- **An increase in the job destruction rate *q* leads to a fall in the wage and in employment (downward shift of the labour demand schedule).**
- **Opposite effect of an increase in marginal productivity *y*.**
- **No involuntary unemployment.**

<u>The efficiency of the competitive equilibrium</u>

- **Risk neutrality**
- **No preference for the present**
- **Social planner maximises sum of instantaneous production inside and outside the market minus labour turnover costs.**
- ***z* is the productivity of a worker outside the market.**
- ***z* has the cumulative distribution function *H*(.).**
- **Planning problem: Find the threshold $\bar{z}$ below which individuals should be employed in the labour market that maximises net aggregate production.**

$$\underset{z}{\text{Max}} \left\{ yNH(z) - C\left[qNH(z)\right] + N\int_{z}^{\infty} x\,dH(x) \right\}$$

Last term represents production outside the market.

<u>FOC</u>:

$$yNH'(z) - qC'\left[qNH(z)\right]NH'(z) - N(1)zH'(z) = 0$$

$$y = qC'\left[qNH(\bar{z})\right] + \bar{z}$$

- **The threshold is equal to the competitive wage according to (2).**
- **The competitive equilibrium is also a social optimum.**
- **This is so even though some are unemployed.**
- **But some people are too productive in home work.**

The Mortensen-Pissarides matching model

- **Imperfect information on the part of job searchers as well as on the part of firms**
- **Matching frictions**

- **Vacant jobs are "urns".**
- **Job applications are "balls" tossed.**
- **A match occurs when a ball goes into an urn.**
- ***D* = number of job seekers**
- ***V* =number of vacancies**
- **Mr i sends simultaneously e_i applications among the *V* vacant jobs.**
- **Employer makes random draw when obtaining more than one application.**

- **Probability of a vacant job receiving an application from Mr i is e_i / V.**
- **Probability of a vacant job <u>not</u> receiving an application from Mr i is $(1 - e_i / V)$.**
- **Probability of a vacant job receiving no application is**

$$\prod_{i=1}^{i=D} \left[1 - (e_i / V)\right]$$

- **Probability of a vacant job receiving at least one application is**

$$1 - \prod_{i=1}^{i=D} \left[1 - (e_i / V)\right].$$

The number of hires, *M*, is given by:

$$M = V\left[1 - \prod_{i=1}^{i=D}(1 - \frac{e_i}{V})\right]$$ 8

Assume that V is large relative to e_i. Then

$$1 - \frac{e_i}{V} \approx e^{-\frac{e_i}{V}}$$

(Same approximation as $\ell n(1-a) \approx -a$ if a is a small number.)

We can write the matching function as:

$$M = M(V, \overline{e}D) = V\left\{1 - e^{-\frac{\overline{e}D}{V}}\right\} \text{if } \overline{e} = \text{average number of applications.}$$

$$\prod_{i=1}^{i=D}(1 - \frac{e_i}{V}) = \prod_{i=1}^{i=D} e^{-\frac{e_i}{V}} = e^{-\frac{e_1}{V}-\frac{e_2}{V}\ldots-\frac{e_D}{V}} = e^{-\frac{\overline{e}D}{V}}$$

One can show that the matching function is

(i) increasing in V and D

(ii) homogenous in V and D of degree 1.

Homogeneity is obvious as $D = kD$ and $V = kV$ gives

$$M = kV\left\{1 - e^{-\frac{\overline{e}D}{V}}\right\}.$$

- $\overline{e}$ **can be regarded as a measure of average search intensity.**
- **Higher $\overline{e}$ increases matching efficiency.**
- **The probability of Mr i finding a job is:** $\dfrac{e_i M(V, \overline{e}D)}{\overline{e}D}$

The probability is larger the higher is the relative search effort $e_i / \overline{e}$.

- **Empirical matching functions are often assumed to be Cobb-Douglas**

$$M = M(V, \overline{e}D) = kV^{a}(\overline{e}D)^{1-a}$$

$$M = kV^{a}U^{1-a}$$

- **CRS is accepted in most empirical studies.**
- **Estimate of 1-α is in the range [0.5, 0.7] with hires of only unemployed and in the range [0.3, 0.4] with all hires.**
- **Matching efficiency deteriorated during and after the Great Recession.**
- **Higher incidence of long-term unemployment is probable explanation.**

Properties of the matching function

- ***M*** **(*V, D*) is the instantaneous flow of hires at date *t.***
- ***M*** **(V, *D*) *dt* is the flow of hires over the interval [*t, t*+*dt*].**
- **$M_V > 0$ and $M_D > 0$.**
- **$M(V, 0) = M(0, D) = 0$.**
- **Only unemployed persons are assumed to search for jobs, such that $D = U$.**
- **CRS**

Probability of filling a vacant job per unit of time:

$$\frac{M(V,\ U)}{V} \ = \ M\left(1, \frac{U}{V}\right) \ \equiv \ m(\theta) \ = \ M\left(1, \frac{1}{\theta}\right) \qquad (3)$$

$\theta = \dfrac{V}{U}$ **is labour market tightness.**

Differentiate (3) w.r.t. to $V/U \ = \ \theta$

$$m'(\theta) \ = \ -M_u\left[1,\ \frac{U}{V}\right]\frac{U^2}{V^2} \ < \ 0$$

Hence a tighter labour market reduces the probability that a vacancy will be filled.

<u>The exit rate from unemployment (the hazard rate)</u>

$$\frac{M(V,U)}{U} = \frac{V}{U}\,\frac{M(V,U)}{V} = \theta m(\theta) \qquad (4)$$

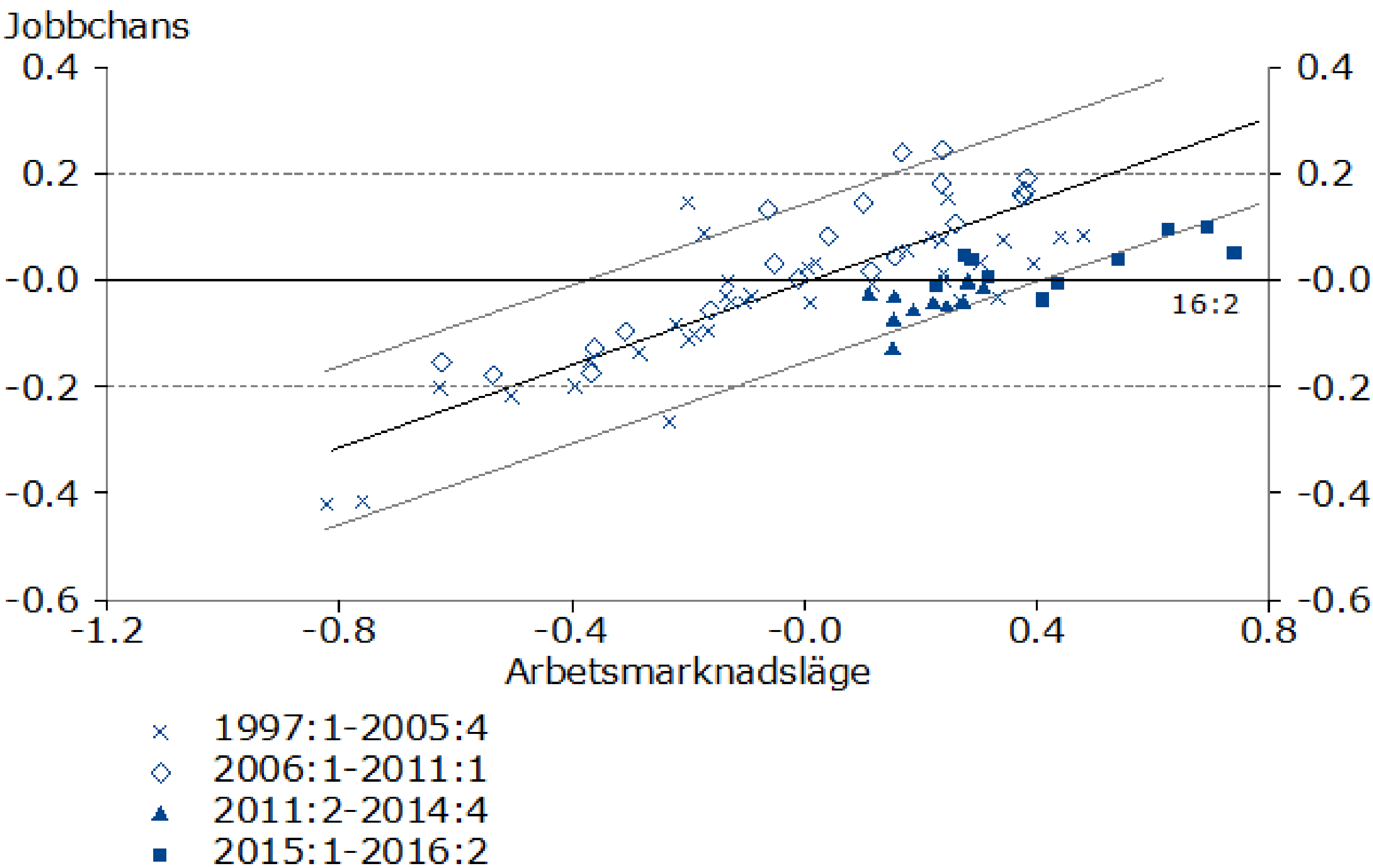
$$\frac{\partial\left[\frac{M(V,U)}{U}\right]}{\partial\left(\frac{V}{U}\right)} = \frac{\partial\left[M\left(\frac{V}{U},1\right)\right]}{\partial\left(\frac{V}{U}\right)} = \frac{\partial\left[M(\theta,1)\right]}{\partial\theta} = M_V(\theta,\ 1) = M_V\left(\frac{V}{U},1\right)$$

Differentiate (4) w.r.t. *V*/*U*!

$$\frac{\partial\left[\theta m(\theta)\right]}{\partial\theta} = m(\theta) + \theta m'(\theta) = M_V(V/U,\ 1) > 0$$

- **The exit rate from unemployment is increasing in labour market tightness.**

Job finding rate and labour market tightness in Sweden



Trading externalities

- **An increase in the number of vacant jobs diminishes the rate at which vacant jobs are filled and increases the exit rate from unemployment.**

- **An increase in the number of unemployed increases the rate at which vacant jobs are filled and reduces the exit rate from unemployment.**

- **Between-group externalities are positive, but within-group externalities are negative**
    - **competition effects**
    - **congestion effects**

Equilibrium flows and the Beveridge curve

***U* = unemployment**
***L* = employment**
***N* = labour force**

$$\dot{U} = \dot{N} + qL - M = \dot{N} + qL - \theta m(\theta)U \qquad (5)$$

$\dot{N} + qL$ **is inflow into unemployment**

$\theta m(\theta)U$ **is hirings = outflow from unemployment**

$n = \dfrac{\dot{N}}{N}$ **= labour force growth rate**

$u = \dfrac{U}{N}$ **= unemployment rate**

Divide (5) by *N*

$$\frac{\dot{U}}{N} = \frac{\dot{N}}{N} + q \cdot \frac{L}{N} - \frac{\theta m(\theta)U}{N}$$

$$\frac{\dot{U}}{N} = n + q\frac{N-U}{N} - \theta m(\theta)u$$

$$\frac{\dot{U}}{N} = n + q(1-u) - \theta m(\theta)u \qquad \text{(A)}$$

We have:

$$\dot{u} = \dot{\left(\frac{U}{N}\right)} = \frac{N\dot{U} - U\dot{N}}{N^2}$$

$$\dot{u}N = \dot{U} - \dot{N}u$$

$$\dot{U} = \dot{N}u - \dot{u}N \qquad \text{(B)}$$

Substitute (B) into (A) and simplify:

$$\dot{u} = q + n - \left[q + n + \theta m(\theta)\right]u$$

We are interested in the steady state with $\dot{u} = 0.$

Then:

$$u = \frac{q + n}{q + n + \theta m(\theta)} \qquad (7)$$

$$\theta = \frac{V}{U} = \frac{\nu}{u} \quad \text{where } \nu = \frac{V}{N}$$

$$u = \frac{q + n}{q + n + \frac{\nu}{u} m\left(\frac{\nu}{u}\right)} \qquad \text{(7A)}$$

- **(7A) defines a relationship between the vacancy rate ν and the unemployment rate *u*.**
- **This is the theoretical derivation of the Beveridge curve.**
- **It can be shown to be downward-sloping and convex.**

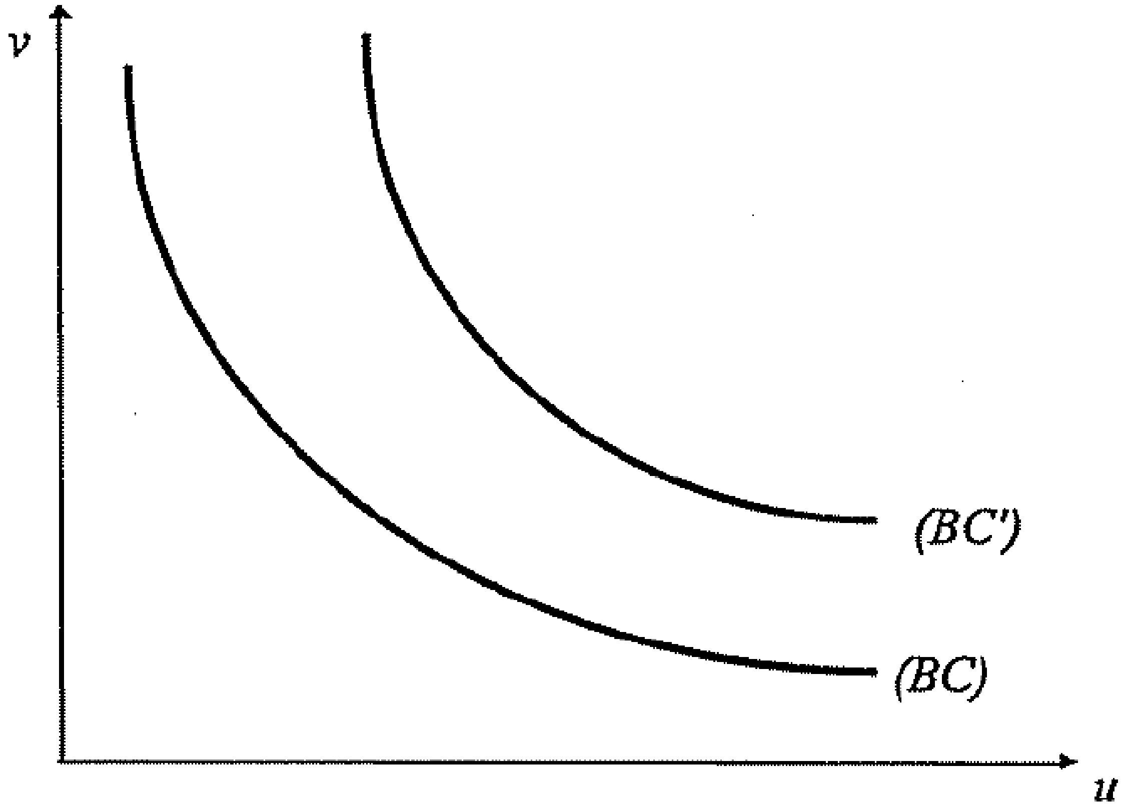
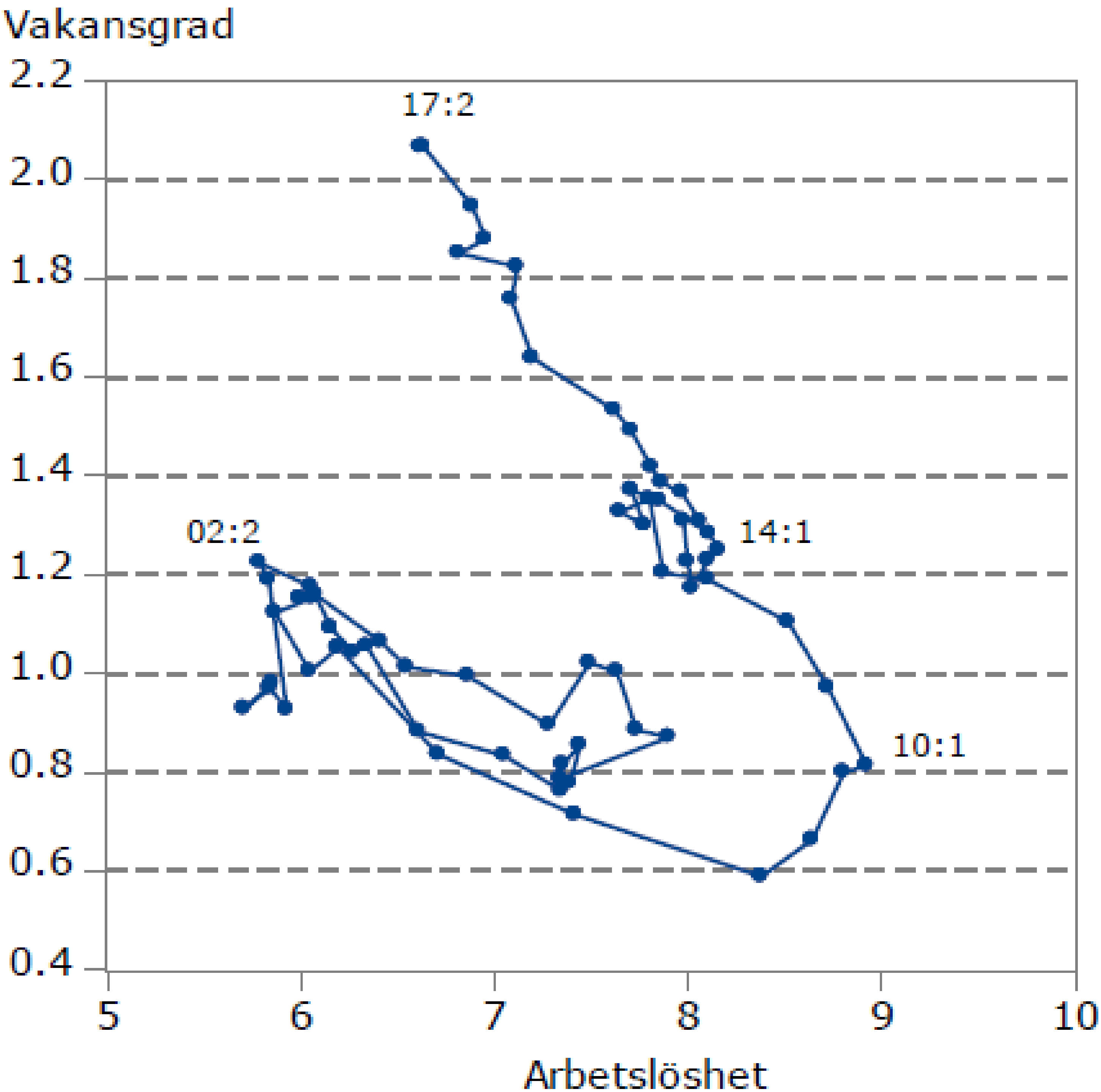


FIGURE 9.1
The Beveridge curve.

The Beveridge curve in Sweden



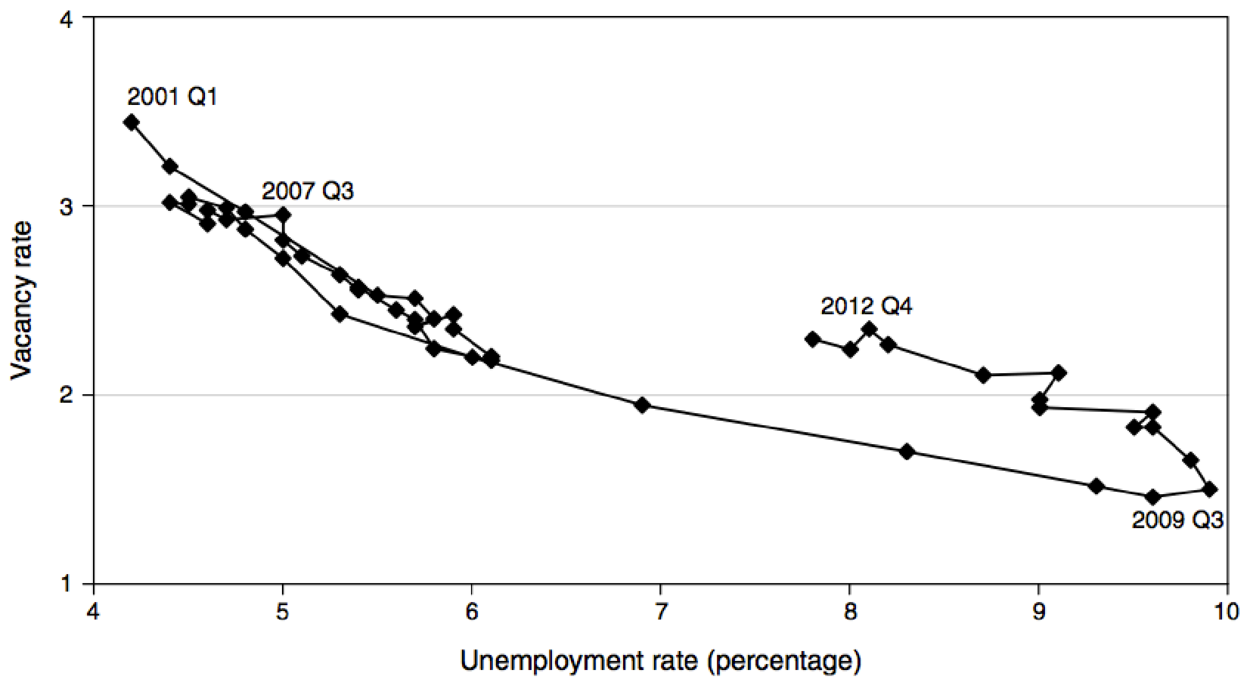


Figure 9.17
The Beveridge curve in the United States, 2001–2012. The vacancy rate is defined as the number of job openings divided by the sum of employment and job openings.

Source: Bureau of Labor Statistics data on openings.

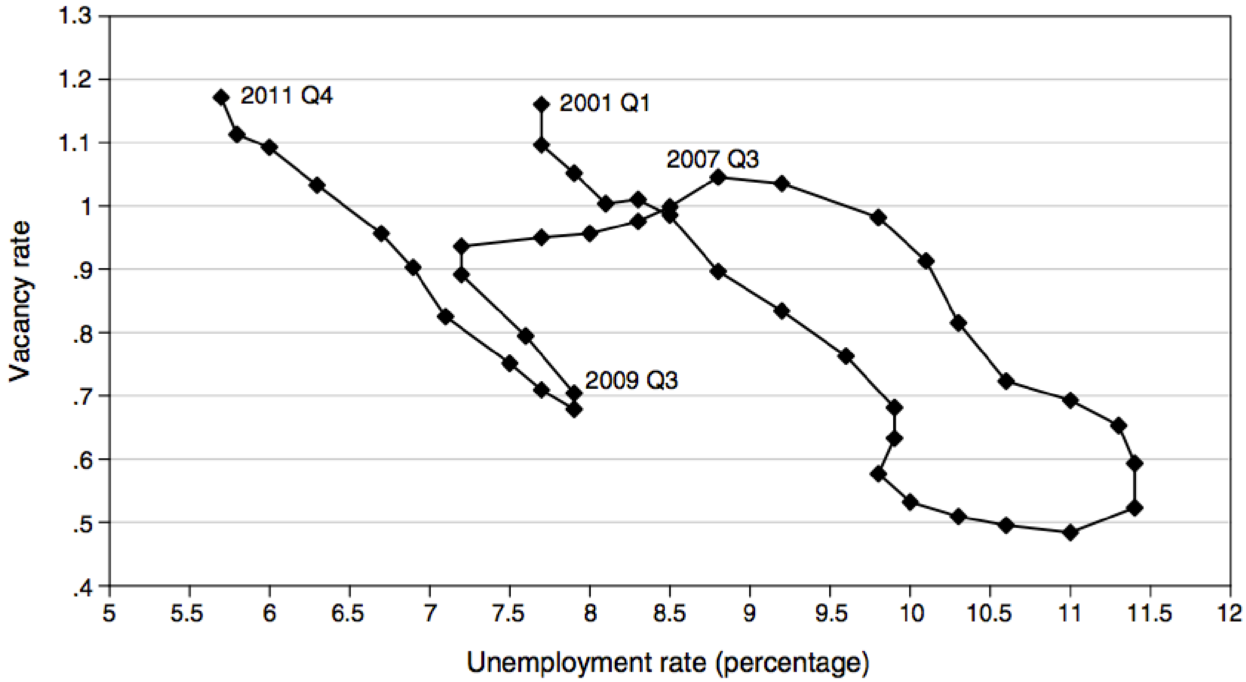


FIGURE 9.18
The Beveridge curve in Germany, 2001–2011.

Source: OECD Main Economic Indicators database and national sources.

The model

- **One good**
- **One production factor: labour**
- **Each firm has one job that can be either filled or vacant.**
- **A filled job produces *y* per unit of time.**

The profit from a filled job

- **In each time interval a filled job may become vacant with probability *qdt*.**
- ***r* = the real interest rate**
- π_e **= the present value of a filled job**
- π_v **= the present value of a vacancy**

$$\pi_e = \frac{1}{1 + rdt}\left[\underbrace{(y-w)dt}_{\text{instantaneous flow of profits}} + \underbrace{qdt\pi_v + (1-qdt)\pi_e}_{\text{expected future profits}}\right]$$

$$r\pi_e = y - w + q(\pi_v - \pi_e) \qquad (9)$$

The return from a filled job is the sum of instantaneous profits plus the expected capital gain (minus the expected capital loss) from the job becoming vacant.

The profit from a vacant job

h = the cost of a vacant job per unit of time

$$\pi_v = \frac{1}{1+rdt}\left\{\underbrace{-hdt}_{\substack{\text{instantaneous}\\ \text{flow of cost}}} + \underbrace{m(\theta)dt\pi_e + \left[1-m(\theta)dt\right]\pi_v}_{\text{expected future profits}}\right\}$$

Rearrange terms and divide by dt:

$$r\pi_v = -h + m(\theta)(\pi_e - \pi_v) \qquad (10)$$

The instantaneous return from a vacancy is minus the cost of a vacancy plus the expected capital gain if the vacancy is filled.

<u>Labour demand</u>

Free-entry-condition: entry of new firms until all profits from a vacancy are wiped out.

$$\pi_v = 0$$

$\pi_v = 0$ in equation (10) gives:

$$\pi_e = \frac{h}{m(\theta)} \qquad \text{(C)}$$

Put $\pi_v = 0$ in equation (9) and solve for π_e :

$$\pi_e = \frac{y-w}{r+q} \qquad \text{(D)}$$

(C) and (D) together give:

$$\frac{h}{m(\theta)} = \frac{y-w}{r+q} \qquad (11)$$

LHS: average cost of a vacant job.

- **"exit rate" from vacancies is $m(\theta)$.**
- **hence average duration of a vacancy is $1/m(\theta)$.**
- **hence average cost of a vacant job is $\left[h \cdot 1/m(\theta)\right]$.**

RHS: expected discounted profit from a filled job.

<u>Interpretation</u>: In a free-entry equilibrium the average cost of a vacant job must equal the profit expected from a filled job.

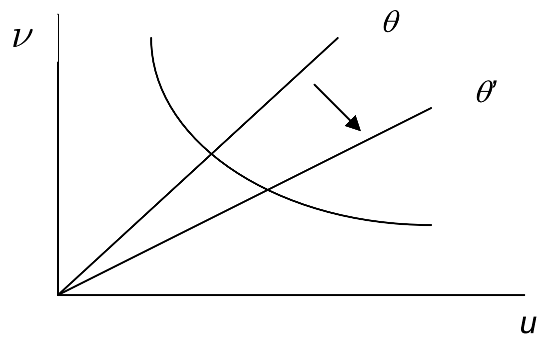
(11) defines a "labour demand schedule": a decreasing relationship between the wage and labour market tightness.

$$\frac{h}{m(\theta)} = \frac{y-w}{r+q}$$

$$w\uparrow \;\Rightarrow\; y-w\downarrow \;\Rightarrow\; \frac{y-w}{r+q}\downarrow \;\Rightarrow\; RHS\downarrow$$

$$\theta\downarrow \;\Rightarrow\; m(\theta)\uparrow \;\Rightarrow\; \frac{h}{m(\theta)}\downarrow \;\Rightarrow\; LHS\downarrow$$

- **If wages are exogenous, unemployment, *u*, and labour market tightness can be solved out from Beveridge curve (equation 7) and labour demand schedule (equation 11).**
- **But more reasonable to assume that wages are bargained over.**



$w\uparrow\Rightarrow\theta\downarrow\Rightarrow u\uparrow$

The behaviour of workers

N individuals in the work force

Infinite life span

V_e **= value of employment**

V_u **= value of unemployment**

q = rate of job destruction

w = real wage

y = output per worker

z = income as unemployed

$\theta m(\theta)$ **= exit rate from unemployment**

Stationary equilibrium

$$rV_e = w + q(V_u - V_e) \qquad (12)$$

$$rV_u = z + \theta m(\theta)(V_e - V_u)$$

Surplus sharing

S = surplus from a match between an employer and a worker

- **The surplus is the sum of rents that a filled job paying w produces**
- **Rent = difference between what the individual gets in a contracted relationship and what the individual would get from the best alternative opportunity**
- **Rent for the employee:** $V_e - V_u$
- **Rent for the employer:** $\pi_e - \pi_V$

S = $V_e - V_u + \pi_e - \pi_V$

$\gamma \in [0,\ 1]$ **is the relative bargaining power of a worker.**

$$V_e - V_u \ = \ \gamma S \qquad (15)$$

$$\pi_e - \pi_V \ = \ (1-\gamma)S$$

This would be the outcome from Nash bargaining:

$$\underset{w}{\text{Max}} \quad (V_e - V_u)^{\gamma}(\pi_e - \pi_V)^{1-\gamma}$$

From earlier equations:

$$S \ = \ \frac{y - r(V_u + \pi_V)}{r \ + \ q} \qquad (17)$$

(9) and (12) can be written:

$$V_e - V_u \ = \ \frac{w - rV_u}{r \ + \ q} \qquad (18)$$

$$\pi_e - \pi_V \ = \ \frac{y - w - r\pi_V}{r \ + \ q}$$

(15), (17) and (18) together give in a free-entry equilibrium with $\pi_V \ = \ 0$:

$$w \ = \ rV_u \ + \ \gamma(y - rV_u) \qquad (19)$$

<u>Interpretation</u>:

- **If unemployed (alternative opportunity), the worker gets the utility flow rV_u = the reservation wage.**
- **On a job, the worker in addition gets a fraction, γ, of the output produced less the reservation wage, rV_u.**

<u>Wage curve</u>

$$rV_u = z + \theta m(\theta)(V_e - V_u) \qquad (13)$$

$$V_e - V_u = \gamma S \qquad (15)$$

These two equations give:

$$rV_u = z + \theta m(\theta)\gamma S$$

Together with:

$$S = \frac{y - r(V_u + \pi_V)}{r + q} = \frac{y - rV_u}{r + q}$$

In a free-entry equilibrium, we have

$$rV_u = \frac{z(r + q) + \gamma y\theta m(\theta)}{r + q) + \gamma\theta m(\theta)}$$

Substitution into wage equation (19) gives:

$$w = z + (y - z)\Gamma(\theta) \quad \text{with} \quad \Gamma(\theta) = \frac{\gamma\left[r + q + \theta m(\theta)\right]}{r + q + \gamma\theta m(\theta)}$$

- **The exit rate from unemployment $\theta m(\theta)$ increases with θ.**
- **Hence $\Gamma'(\theta) > 0$.**
- **Higher labour market tightness θ increases the wage**

  - better outside opportunity

- $\partial\Gamma / \partial q < 0; q \uparrow$ **means a smaller surplus to share**

- **The relationship between *w* and θ is a <u>wage curve</u>**
  - **for given ν, it defines a negative relationship between w and u (positive between the wage and employment).**

<u>Empirical results</u>

Workers appropriate 30 per cent of the rents, i.e. $\gamma \approx 0.3$.

<u>Equilibrium labour market tightness</u>

Eliminate *w* between (11) and (20)

$$\frac{h}{m(\theta)} = \frac{y-w}{r+q} \qquad \text{(11): labour demand}$$

$$w = z + (y-z)\Gamma(\theta) \qquad \text{(20): wage setting}$$

We get:

$$\frac{(1-\gamma)(y-z)}{r+q+\gamma\theta m(\theta)} = \frac{h}{m(\theta)} \qquad (21)$$

Comparative statistics can be made by differentiating equation (21) totally.

Knowing θ **from (21), we get unemployment from the Beveridge curve (7).**

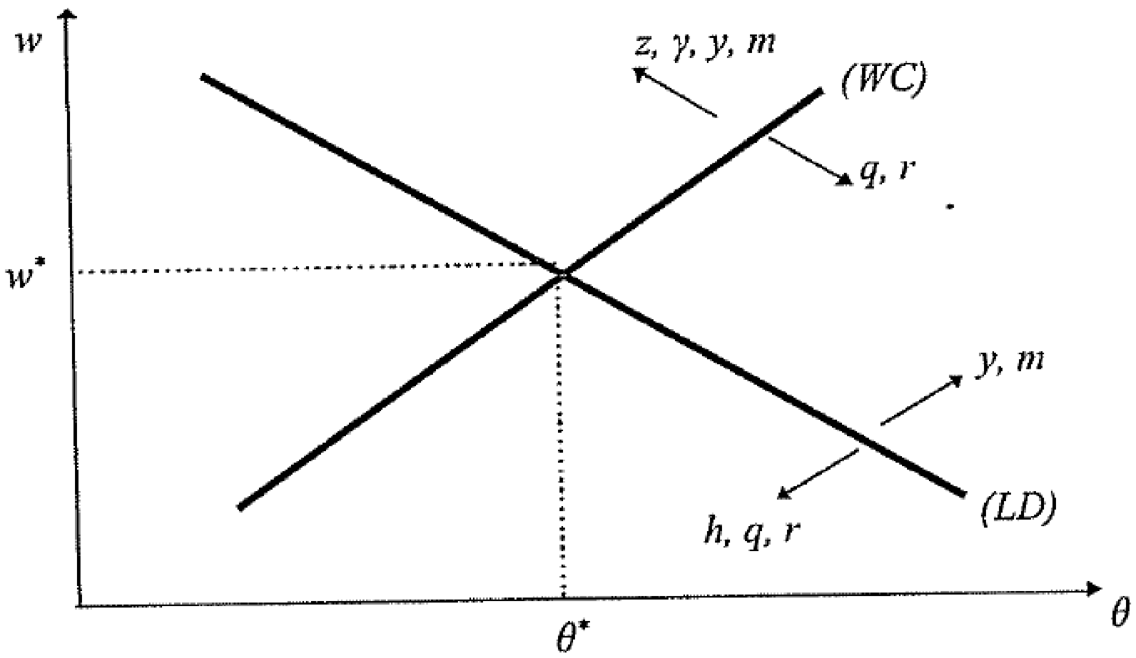


FIGURE 9.4
The negotiated wage and labor market tightness.

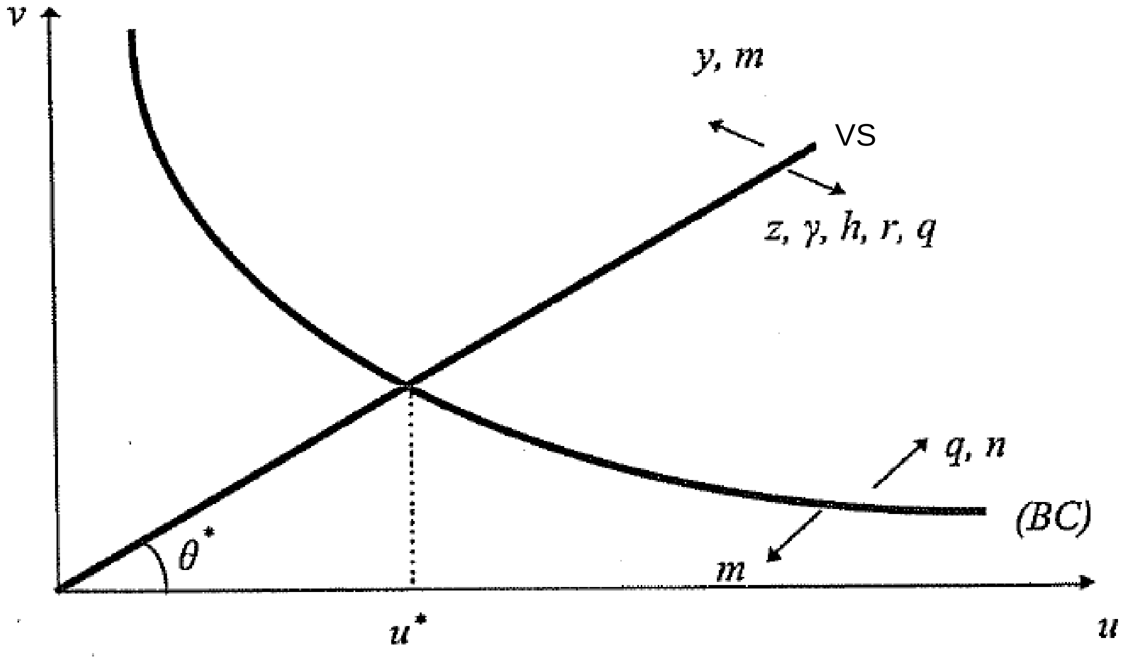


FIGURE 9.5
Vacant jobs and unemployment.

Table 9.8

Comparative statics of stationary equilibrium.

	z	γ	h	m	y	q	r	n
w	+	+	−	+	+	−	−	0
θ	−	−	−	+	+	−	−	0
u	+	+	+	−	−	+	+	+

<u>Higher growth of the labour force</u> (*n*)

- ***WC* and *LD* curves are unchanged.**
- ***VS* curve is unchanged.**
- **Beveridge curve is shifted to the right.**
- ***W* and θ are unchanged.**
- $u \uparrow$
- **This is equivalent to a deterioration of the matching process.**

<u>Increased bargaining power for workers</u> (γ)

- ***LD* unchanged.**
- ***WC* is shifted upwards.**
- $W \uparrow \theta \downarrow$
- ***VS* curve rotates down.**
- **Beveridge curve is unchanged.**
- $u \uparrow$

<u>Increased unemployment benefits</u> (*z*)

- **Similar effect as increase in bargaining power**

<u>Increased productivity</u> (*y*)

- **Both *WC* and *LD* are shifted upwards**
  - **larger pie to share**
  - **tendency to higher wage.**
- $w \uparrow$
- **Opposing effects on θ, but net effect is $\theta \uparrow$.**
- ***VS* curve rotates up.**
- **Beveridge curve is unchanged.**
- $u \downarrow$
- **Important assumption: *z* and *h* are independent of *y*.**
- **If $z = z'w$ and $h = h'w$, so that unemployment benefits and hiring costs are perfectly indexed to the wage, then θ and *u* are unaffected by *y*.**

<u>Interpretation</u>: The productivity level affects unemployment in the short run, but not in the long run.

<u>Increased efficiency of the matching process</u>

- **Multiply matching function $m(\cdot)$ with a constant larger than unity.**
- **Increased probability of returning to work $\Gamma(\theta)\uparrow$: *WC* curve shifts upwards.**
- **Firms offer more jobs for a given wage as the profitability if opening vacancies increases: *LD* curve shifts to the right.**
- **$w\uparrow$; opposing effects on θ, but net effect is $\theta\uparrow$.**
- ***VS* curve rotates upwards at the same time as the Beveridge curve shifts downwards: hence $u\downarrow$.**

<u>Increased job destruction rate</u> (*q*)

- **Equivalent to a reduction in matching efficiency.**

<u>An increase in the interest rate</u> (*r*)

- **The discounted value of future profits falls: lower incentive to post vacancies.**
- ***LD* curve shifts down.**
- **But *WC* curve also shifts down.**
- **$w\downarrow$; opposing effects on θ. Net fall in θ.**
- ***VS* curve rotates downwards: $u\uparrow$.**

Table 9.9

Parameter values for the matching model.

γ	h	q	r	n
0.5	0.3	0.15	0.05	0.01

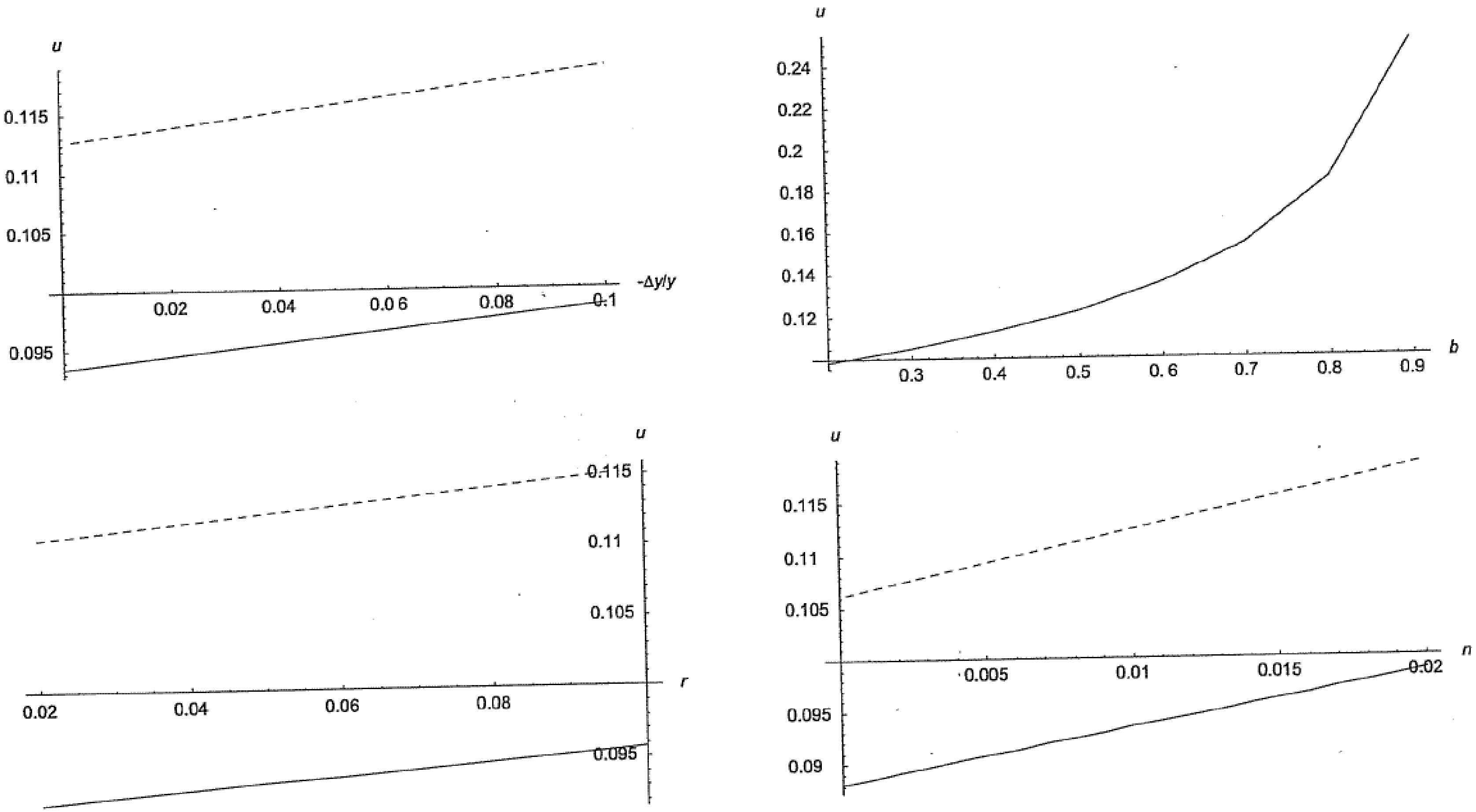


FIGURE 9.6
Simulations on the basis of the matching model.
Solid line: $b = 0.1$; dashed line: $b = 0.4$.

Unemployment volatility puzzle

- Shimer (2005) found that the matching model could not explain real-world unemployment volatility
  - too small unemployment variation
  - too large real wage variation
- Possible solutions of the puzzle
  - high value of non-market activity and low bargaining power
  - wage rigidities
  - flexible wages for hiring wages but not for continuing wages (Pissarides 2009) as well as large hiring costs

Efficiency of labour market equilibrium

- **Both positive and negative externalities in the matching process**
  - positive externalities between groups
  - negative externalities within groups (congestion effects)

- **A larger number of vacancies**
  - lower probability to fill each vacancy
  - higher probability to find a job for each unemployed person

- **A larger number of unemployed persons**
  - lower probability to find a job for each unemployed person
  - higher probability to fill each vacancy

- **A social planner would take all the externalities into account**

- **Decentralised equilibrium needs not coincide with social optimum as the externalities are not taken into account**
  - but since externalities go in opposite directions the decentralised equilibrium could coincide with the social optimum

<u>Social optimum</u>

No discounting $\Leftrightarrow r = 0$

Constant labour force $\Leftrightarrow n = 0$

Social welfare is Ω

$$\Omega = \mathrm{y}L + zU - hV$$

z **= returns on leisure and home production**

$\omega = \Omega / N$ **= total income per capita**

$$\frac{\Omega}{N} = y \cdot \frac{L}{N} + z \cdot \frac{U}{N} - h\frac{V}{N}$$

$$N = L + U$$

$$1 = \frac{L}{N} + \frac{U}{N}$$

$$1 = l + u$$

$$\omega = y(1-u) + zu - h\nu$$

But since $\theta = \dfrac{\nu}{u}$, we have $\nu = \theta u$

$$\because \omega = y(1-u) + zu - h\theta u$$

The general formulation of the Beveridge curve:

$$u = \frac{q + n}{q + n + \theta m(\theta)}$$

$$n = 0 \Rightarrow u = \frac{q}{q + \theta m(\theta)}$$

The optimisation problem of the social planner:

$$\underset{\theta,\, u}{\text{Max}} \quad \omega = y(1-u) + zu - h\theta u$$

$$\text{s.t.} \quad u = \frac{q}{q + \theta m(\theta)}$$

FOC

$$\frac{\partial \omega}{\partial \theta} = -y\frac{\partial u}{\partial \theta} + z\frac{\partial u}{\partial \theta} - hu - h\theta\frac{\partial u}{\partial \theta} = 0$$

$$\frac{\partial u}{\partial \theta} = \frac{-\left\{\theta m'(\theta) + m(\theta)\right\}q}{\left[q + \theta m(\theta)\right]^2}$$

$$\frac{\partial u}{\partial \theta}\left[z - y - h\theta\right] = hu$$

$$\frac{-\left\{\theta m'(\theta) + m(\theta)\right\}q}{\left[q + \theta m(\theta)\right]^2}\left[z - y - h\theta\right] = hu$$

Define $\eta(\theta) = -\dfrac{\theta m'(\theta)}{m(\theta)}$

$$\frac{-m(\theta)\left\{\frac{\theta m'(\theta)}{m(\theta)} + 1\right\}q}{\left[q + \theta m(\theta)\right]^2}\left[z - y - h\theta\right] = hu = h\frac{q}{q + \theta m(\theta)}$$

$$\frac{-m(\theta)\left\{-\eta(\theta) + 1\right\}q}{\left[q + \theta m(\theta)\right]^2}\left[z - y - h\theta\right] = h\frac{q}{q + \theta m(\theta)}$$

$$\frac{-m(\theta)\left\{-\eta(\theta) + 1\right\}}{\left[q + \theta m(\theta)\right]}\left[z - y - h\theta\right] = h$$

$$\frac{-m(\theta)\left[1 - \eta(\theta)\right]}{q + \theta m(\theta)}\left[z - y\right] + \frac{h\theta m(\theta)\left[1\text{-}\eta(\theta)\right]}{q + \theta m(\theta)} = h$$

$$\frac{m(\theta)\left[1 - \eta(\theta)\right]\left[y - z\right]}{q + \theta m(\theta)} = h\frac{\left[q + \theta m(\theta) - \theta m(\theta) + \theta m(\theta)\eta(\theta)\right]}{q + \theta m(\theta)}$$

$$\frac{\left[1 - \eta(\theta)\right]\left[y - z\right]}{q + \theta m(\theta)\eta(\theta)} = \frac{h}{m(\theta)} \qquad (49)$$

(49) defines the social optimum.

Compare (49) with equation (21) for the decentralised equilibrium:

$$\frac{(1-\gamma)(y-z)}{r + q + \gamma\theta m(\theta)} = \frac{h}{m(\theta)} \qquad (21)$$

$$r = 0 \Rightarrow \frac{(1-\gamma)(y-z)}{q + \gamma\theta m(\theta)} = \frac{h}{m(\theta)} \qquad (21')$$

- **(49) and (21) coincide if** $\eta(\theta) = \gamma$**.**
- **The decentralised equilibrium is socially efficient if the bargaining power of workers equals the elasticity of the matching function w.r.t. to unemployment.**
- **This is known as the Hosios condition.**
- $\eta(\theta) = \gamma$ **gives the right blend of congestion effects and positive externalities.**

$$\eta(\theta) = -\frac{\theta m'(\theta)}{m(\theta)}$$

$$m(\theta) = \frac{M(V,\ U)}{V} = M\left(1,\ \frac{U}{V}\right) = M\left(1,\ \frac{1}{\theta}\right)$$

$$\frac{\partial m}{\partial \theta} = -\frac{M_u}{\theta^2}$$

$$\eta(\theta) = -\left(-\frac{M_u}{\theta^2}\right)\frac{\theta}{M\left(1,\ \frac{1}{\theta}\right)} = \frac{M_u}{\theta M\left(1,\ \frac{1}{\theta}\right)} = \frac{M_u}{\frac{V}{U} M\left(1,\ \frac{U}{V}\right)} =$$

$$\frac{M_u}{\frac{1}{U} M(V,\ U)} = \frac{M_u U}{M(V,\ U)} = \frac{\partial M}{\partial U} \cdot \frac{U}{M}$$